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Neighborhoods of Certain Classes of Analytic Functions Defined by Normalized Function $az^2J''_{\vartheta}(z) + bzJ'_{\vartheta}(z) + cJ_{\vartheta}(z)$

Murat Çağlar[a], Erhan Deniz[b], Sercan Kazımoğlu[c]

[a]*Department of Mathematics, Faculty of Science and Letters, Kafkas University, Kars, Turkey*
[b]*Department of Mathematics, Faculty of Science and Letters, Kafkas University, Kars, Turkey*
[c]*Department of Mathematics, Faculty of Science and Letters, Kafkas University, Kars, Turkey*

Abstract. In this paper, we introduce a new subclass of analytic functions in the open unit disk $\mathcal{U}$ with negative coefficients defined by normalized of the $az^2J''_{\vartheta}(z) + bzJ'_{\vartheta}(z) + cJ_{\vartheta}(z)$ function, where $J_{\vartheta}(z)$ is called the Bessel function of the first kind of order ϑ. The object of the present paper is to determine coefficient inequalities, inclusion relations and neighborhoods properties for functions $f(z)$ belonging to this subclass.

1. Introduction

Let $\mathcal{A}$ be a class of functions f of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \tag{1}$$

that are analytic in the open unit disk $\mathcal{U} = \{z : |z| < 1\}$. Denote by $\mathcal{A}(n)$ the class of functions consisting of functions f of the form

$$f(z) = z - \sum_{n=2}^{\infty} a_n z^n \tag{2}$$

which are analytic in $\mathcal{U}$.
We recall that the convolution (or Hadamard product) of two functions

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \quad \text{and} \quad g(z) = z + \sum_{n=2}^{\infty} b_n z^n$$

is given by

$$(f * g)(z) := z + \sum_{n=2}^{\infty} a_n b_n z^n =: (g * f)(z) \quad (z \in \mathcal{U}).$$

Corresponding author: MÇ mail address: mcaglar25@gmail.com ORCID:0000-0001-8147-0343, ED ORCID:0000-0002-9570-8583 SK ORCID:0000-0002-1023-4500
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Note that $f * g \in \mathcal{A}$.
Next, following the earlier investigations by Goodman [8], Ruscheweyh [16], Silverman [18] and Altıntaş et al. [1, 2] (see also [4]-[7], [10], [12], [14]-[16]), we define the $(n, \delta)-$neighborhood of a function $f \in \mathcal{A}(n)$ by

$$\mathcal{N}_{n,\delta}(f) = \left\{ g \in \mathcal{A}(n) : g(z) = z - \sum_{n=2}^{\infty} b_n z^n \text{ and } \sum_{n=2}^{\infty} n |a_n - b_n| \le \delta \right\}. \quad (3)$$

For $e(z) = z$, we have

$$\mathcal{N}_{n,\delta}(e) = \left\{ g \in \mathcal{A}(n) : g(z) = z - \sum_{n=2}^{\infty} b_n z^n \text{ and } \sum_{n=2}^{\infty} n |b_n| \le \delta \right\}. \quad (4)$$

A function $f \in \mathcal{A}(n)$ is $\alpha-$starlike of complex order γ, denoted by $f \in \mathcal{S}_n^*(\beta, \gamma)$ if it satisfies the following condition

$$\Re\left\{1 + \frac{1}{\gamma}\left(\frac{zf'(z)}{f(z)} - 1\right)\right\} > \beta \qquad (\gamma \in \mathbb{C}\setminus\{0\}, 0 \le \beta < 1, z \in \mathcal{U}),$$

and a function $f \in \mathcal{A}(n)$ is $\beta-$convex of complex order γ, denoted by $f \in C_n(\beta, \gamma)$ if it satisfies the following condition

$$\Re\left\{1 + \frac{1}{\gamma}\frac{zf''(z)}{f'(z)}\right\} > \beta \qquad (\gamma \in \mathbb{C}\setminus\{0\}, 0 \le \beta < 1, z \in \mathcal{U}).$$

The Bessel function of the first kind of order ϑ is defined by [13, p.217]

$$J_\vartheta(z) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!\Gamma(n + \vartheta + 1)}\left(\frac{z}{2}\right)^{2n+\vartheta} \qquad (z \in \mathbb{C}). \quad (5)$$

We know that it has all its zeros real for $\vartheta > -1$. Here now we consider mainly the general function

$$N_\vartheta(z) = az^2 J''_\vartheta(z) + bzJ'_\vartheta(z) + cJ_\vartheta(z)$$

studied by Mercer [11]. Here, as in [11], $q = b - a$ and

$$(c = 0 \text{ and } q \neq 0) \ \ or \ \ (c > 0 \text{ and } q > 0).$$

From (5), we have the power series representation

$$N_\vartheta(z) = \sum_{n=0}^{\infty} \frac{Q(2n + \vartheta)(-1)^n}{n!\Gamma(n + \vartheta + 1)}\left(\frac{z}{2}\right)^{2n+\vartheta} \qquad (z \in \mathbb{C}) \quad (6)$$

where $Q(\vartheta) = a\vartheta(v - 1) + b\vartheta + c \ \ (a, b, c \in \mathbb{R})$. Lastly, Baricz, Çağlar and Deniz [3] obtained sufficient and necessary conditions for the starlikeness of a normalized form of N_ϑ by using results of Mercer [11], Ismail and Muldoon [9] and Shah and Trimble [17].
Note that N_ϑ is not belong to the class $\mathcal{A}$. Therefore, we consider the following normalization for the function $N_\vartheta(z)$:

$$\tilde{N}_\vartheta(z) = \frac{2^\vartheta\Gamma(\vartheta + 1)z^{1-\frac{\vartheta}{2}}}{Q(\vartheta)}N_\vartheta\left(\sqrt{z}\right). \quad (7)$$

In the rest of this paper, the quadratic $Q(\vartheta) = a\vartheta(\vartheta - 1) + b\vartheta + c$ will always provide on $(a, b, c \in \mathbb{R})$ $(c = 0 \text{ and } q \neq 0) \ or (c > 0 \text{ and } q > 0)$. Moreover, ϑ_0 is the largest real root of the quadratic $Q(\vartheta)$ defined according to the above conditions.
Easily, we can write

$$\tilde{N}_\vartheta(z) = z + \sum_{n=2}^{\infty} \frac{(-1)^{n+1}\Gamma(\vartheta + 1)Q(\vartheta + 2(n - 1))}{4^{n-1}(n - 1)!\Gamma(\vartheta + n)Q(\vartheta)}z^n \qquad (z \in \mathcal{U}). \quad (8)$$

In terms of Hadamard product and $\tilde{N}_{\vartheta}(z)$ given by (8), a new operator $\tilde{N}_{\vartheta}: \mathcal{A} \to \mathcal{A}$ can be defined as follows:

$$\tilde{N}_{\vartheta} f(z) = \left(\tilde{N}_{\vartheta} * f\right)(z) = z + \sum_{n=2}^{\infty} \frac{(-1)^{n+1}\Gamma(\vartheta+1)\,Q(\vartheta+2(n-1))}{4^{n-1}(n-1)!\Gamma(\vartheta+n)\,Q(\vartheta)} a_n z^n \quad (z \in \mathcal{U}). \tag{9}$$

If $f \in \mathcal{A}(n)$ is given by (2) then we have

$$\tilde{N}_{\vartheta} f(z) = z - \sum_{n=2}^{\infty} \frac{(-1)^{n+1}\Gamma(\vartheta+1)\,Q(\vartheta+2(n-1))}{4^{n-1}(n-1)!\Gamma(\vartheta+n)\,Q(\vartheta)} a_n z^n \quad (z \in \mathcal{U}). \tag{10}$$

Finally, by using the differential operator defined by (10), we investigate the subclasses $\mathcal{M}_{\vartheta}^{n}(\beta,\gamma)$ and $\mathcal{R}_{\vartheta}^{n}(\beta,\gamma,\mu)$ of $\mathcal{A}(n)$ consisting of functions f as following:

Definition 1.1. *The subclass $\mathcal{M}_{\vartheta}^{n}(\beta,\gamma)$ of $\mathcal{A}(n)$ is defined as the class of functions f such that*

$$\left|\frac{1}{\gamma}\left(\frac{z\left[\tilde{N}_{\vartheta} f(z)\right]'}{\tilde{N}_{\vartheta} f(z)} - 1\right)\right| < \beta \qquad (z \in \mathcal{U}) \tag{11}$$

where $\gamma \in \mathbb{C}\setminus\{0\}$ and $0 \le \beta < 1$.

Definition 1.2. *Let $\mathcal{R}_{\vartheta}^{n}(\beta,\gamma,\mu)$ denote the subclass of $\mathcal{A}(n)$ consisting of f which satisfy the inequality*

$$\left|\frac{1}{\gamma}\left[(1-\mu)\frac{\tilde{N}_{\vartheta} f(z)}{z} + \mu\left(\tilde{N}_{\vartheta} f(z)\right)' - 1\right]\right| < \beta \qquad (z \in \mathcal{U}) \tag{12}$$

where $\gamma \in \mathbb{C}\setminus\{0\}$ and $0 \le \beta < 1$, $0 \le \mu \le 1$.

In this paper, we obtain the coefficient inequalities, inclusion relations and neighborhood properties of the subclasses $\mathcal{M}_{\vartheta}^{n}(\beta,\gamma)$ and $\mathcal{R}_{\vartheta}^{n}(\beta,\gamma,\mu)$.

2. Coefficient inequalities for the classes $\mathcal{M}_{\vartheta}^{n}(\beta,\gamma)$ and $\mathcal{R}_{\vartheta}^{n}(\beta,\gamma,\mu)$

Theorem 2.1. *Let $f \in \mathcal{A}(n)$. Then $f \in \mathcal{M}_{\vartheta}^{n}(\beta,\gamma)$ if and only if*

$$\sum_{n=2}^{\infty} \frac{(-1)^{n+1}\Gamma(\vartheta+1)Q(\vartheta+2(n-1))}{4^{n-1}(n-1)!\Gamma(\vartheta+n)\,Q(\vartheta)}\left[n-1+\beta\left|\gamma\right|\right] a_n \le \beta\left|\gamma\right| \tag{13}$$

where $\gamma \in \mathbb{C}\setminus\{0\}$ and $0 \le \beta < 1$.

Proof. Let $f \in \mathcal{A}(n)$. Then, by (11) we can write

$$\Re\left\{\frac{z\left[\tilde{N}_{\vartheta} f(z)\right]'}{\tilde{N}_{\vartheta} f(z)} - 1\right\} > -\beta\left|\gamma\right| \qquad (z \in \mathcal{U}). \tag{14}$$

Using (2) and (10), we have,

$$\Re\left\{\frac{-\sum\limits_{n=2}^{\infty} \frac{(-1)^{n+1}\Gamma(\vartheta+1)Q(\vartheta+2(n-1))}{4^{n-1}(n-1)!\Gamma(\vartheta+n)Q(\vartheta)}\left[n-1\right] a_n z^n}{z - \sum\limits_{n=2}^{\infty} \frac{(-1)^{n+1}\Gamma(\vartheta+1)Q(\vartheta+2(n-1))}{4^{n-1}(n-1)!\Gamma(\vartheta+n)Q(\vartheta)} a_n z^n}\right\} > -\beta\left|\gamma\right| \qquad (z \in \mathcal{U}). \tag{15}$$

Since (15) is true for all $z \in \mathcal{U}$, choose values of z on the real axis. Letting $z \to 1$, through the real values, the inequality (15) yields the desired inequality

$$\sum_{n=2}^{\infty} \frac{(-1)^{n+1}\Gamma(\vartheta+1)Q(\vartheta+2(n-1))}{4^{n-1}(n-1)!\Gamma(\vartheta+n)Q(\vartheta)} \left[n-1+\beta\left|\gamma\right|\right] a_n \leq \beta\left|\gamma\right|.$$

Conversely, supposed that inequality (13) holds true and $|z|=1$, we obtain

$$\begin{aligned}\left|\frac{z\left[\Psi_{\lambda,\mu}f(z)\right]'}{\Psi_{\lambda,\mu}f(z)}-1\right| &\leq \left|\frac{\sum_{n=2}^{\infty}\frac{(-1)^{n+1}\Gamma(\vartheta+1)Q(\vartheta+2(n-1))}{4^{n-1}(n-1)!\Gamma(\vartheta+n)Q(\vartheta)}[n-1]a_n z^n}{z-\sum_{n=2}^{\infty}\frac{(-1)^{n+1}\Gamma(\vartheta+1)Q(\vartheta+2(n-1))}{4^{n-1}(n-1)!\Gamma(\vartheta+n)Q(\vartheta)}a_n z^n}\right| \\ &\leq \frac{\sum_{n=2}^{\infty}\frac{(-1)^{n+1}\Gamma(\vartheta+1)Q(\vartheta+2(n-1))}{4^{n-1}(n-1)!\Gamma(\vartheta+n)Q(\vartheta)}[n-1]a_n}{1-\sum_{n=2}^{\infty}\frac{(-1)^{n+1}\Gamma(\vartheta+1)Q(\vartheta+2(n-1))}{4^{n-1}(n-1)!\Gamma(\vartheta+n)Q(\vartheta)}a_n} \\ &\leq \beta\left|\gamma\right|.\end{aligned}$$

Hence, by the maximum modulus theorem, we have $f(z) \in \mathcal{M}_{\vartheta}^{n}(\beta,\gamma)$, which establishes the required result. □

Theorem 2.2. *Let $f \in \mathcal{A}(n)$. Then $f \in \mathcal{R}_{\vartheta}^{n}(\beta,\gamma,\mu)$ if and only if*

$$\sum_{n=2}^{\infty} \frac{(-1)^{n+1}\Gamma(\vartheta+1)Q(\vartheta+2(n-1))}{4^{n-1}(n-1)!\Gamma(\vartheta+n)Q(\vartheta)} \left[1+\mu(n-1)\right] a_n \leq \beta\left|\gamma\right| \tag{16}$$

for $\gamma \in \mathbb{C}\setminus\{0\}$, $0 \leq \beta < 1$ and $0 \leq \mu \leq 1$.

Proof. We omit the proofs since it is similar to Theorem 2.1. □

3. Inclusion relations involving $\mathcal{N}_{n,\delta}(e)$ of the classes $\mathcal{M}_{\vartheta}^{n}(\beta,\gamma)$ and $\mathcal{R}_{\vartheta}^{n}(\beta,\gamma,\mu)$

Theorem 3.1. *If*

$$\delta = \frac{-8\beta\left|\gamma\right|\Gamma(\vartheta+2)Q(\vartheta)}{\left(1+\beta\left|\gamma\right|\right)\Gamma(\vartheta+1)Q(\vartheta+2)} \qquad \left(\left|\gamma\right|<1\right) \tag{17}$$

then $\mathcal{M}_{\vartheta}^{n}(\beta,\gamma) \subset \mathcal{N}_{n,\delta}(e)$.

Proof. Let $f(z) \in \mathcal{M}_{\vartheta}^{n}(\beta,\gamma)$. By Theorem 2.1, we have

$$\frac{-\Gamma(\vartheta+1)Q(\vartheta+2)}{4\Gamma(\vartheta+2)Q(\vartheta)}(1+\beta\left|\gamma\right|)\sum_{n=2}^{\infty}a_n \leq \beta\left|\gamma\right|,$$

which implies

$$\sum_{n=2}^{\infty} a_n \leq \frac{\beta\left|\gamma\right|}{\frac{-\Gamma(\vartheta+1)Q(\vartheta+2)}{4\Gamma(\vartheta+2)Q(\vartheta)}(1+\beta\left|\gamma\right|)}. \tag{18}$$

Using (13) and (18), we get

$$\begin{aligned}\frac{-\Gamma(\vartheta+1)Q(\vartheta+2)}{4\Gamma(\vartheta+2)Q(\vartheta)}\sum_{n=2}^{\infty}na_n &\leq \beta\left|\gamma\right| + \frac{-\Gamma(\vartheta+1)Q(\vartheta+2)}{4\Gamma(\vartheta+2)Q(\vartheta)}(1-\beta\left|\gamma\right|)\sum_{n=2}^{\infty}a_n \\ &\leq \frac{2\beta\left|\gamma\right|}{1+\beta\left|\gamma\right|} = \delta.\end{aligned}$$

That is,

$$\sum_{n=2}^{\infty} na_n \leq \frac{-8\beta\left|\gamma\right|\Gamma\left(\vartheta+2\right)Q(\vartheta)}{\left(1+\beta\left|\gamma\right|\right)\Gamma(\vartheta+1)Q(\vartheta+2)} = \delta.$$

Thus, by the definition given by (4), $f(z) \in \mathcal{N}_{n,\delta}(e)$, which completes the proof. □

Theorem 3.2. *If*

$$\delta = \frac{-8\beta\left|\gamma\right|\Gamma\left(\vartheta+2\right)Q(\vartheta)}{\left(1+\mu\right)\Gamma(\vartheta+1)Q(\vartheta+2)} \quad \left(\left|\gamma\right|<1\right) \tag{19}$$

then $\mathcal{R}_{\vartheta}^{n}(\beta,\gamma,\mu) \subset \mathcal{N}_{n,\delta}\left(e\right)$.

Proof. For $f(z) \in \mathcal{R}_{\vartheta}^{n}(\beta,\gamma,\mu)$ and making use of the condition (16), we obtain

$$\frac{-\Gamma(\vartheta+1)Q(\vartheta+2)}{4\Gamma\left(\vartheta+2\right)Q(\vartheta)}(1+\mu)\sum_{n=2}^{\infty} a_n \leq \beta\left|\gamma\right|$$

so that

$$\sum_{n=2}^{\infty} a_n \leq \frac{-4\beta\left|\gamma\right|\Gamma\left(\vartheta+2\right)Q(\vartheta)}{(1+\mu)\Gamma(\vartheta+1)Q(\vartheta+2)}. \tag{20}$$

Thus, using (16) along with (20), we also get

$$\begin{aligned} -\mu\frac{\Gamma(\vartheta+1)Q(\vartheta+2)}{4\Gamma\left(\vartheta+2\right)Q(\vartheta)}\sum_{n=2}^{\infty} na_n &\leq \beta\left|\gamma\right| + (1-\mu)\frac{\Gamma(\vartheta+1)Q(\vartheta+2)}{4\Gamma\left(\vartheta+2\right)Q(\vartheta)}\sum_{n=2}^{\infty} a_n \\ &\leq \beta\left|\gamma\right| + (\mu-1)\frac{\beta\left|\gamma\right|}{1+\mu}. \end{aligned}$$

Hence,

$$\sum_{n=2}^{\infty} na_n \leq \frac{-8\beta\left|\gamma\right|\Gamma\left(\vartheta+2\right)Q(\vartheta)}{(1+\mu)\Gamma(\vartheta+1)Q(\vartheta+2)} = \delta$$

which in view of (4), completes the proof of theorem. □

4. Neighborhood properties for the classes $\mathcal{M}_{\vartheta}^{n}(\beta,\gamma)$ and $\mathcal{R}_{\vartheta}^{n}(\beta,\gamma,\mu)$

Definition 4.1. *For* $0 \leq \eta < 1$ *and* $z \in \mathcal{U}$, *a function* $f(z) \in \mathcal{M}_{\lambda,\mu}^{n}(\alpha,\gamma)$ *if there exists a function* $g(z) \in \mathcal{M}_{\vartheta}^{n}(\beta,\gamma)$ *such that*

$$\left|\frac{f(z)}{g(z)}-1\right| < 1-\eta. \tag{21}$$

For $0 \leq \eta < 1$ and $z \in \mathcal{U}$, a function $f(z) \in \mathcal{R}_{\vartheta}^{n}(\beta,\gamma,\mu)$ if there exists a function $g\left(z\right) \in \mathcal{R}_{\vartheta}^{n}\left(\beta,\gamma,\mu\right)$ such that the inequality (21) holds true.

Theorem 4.2. *If* $g(z) \in \mathcal{M}_{\vartheta}^{n}(\beta,\gamma)$ *and*

$$\eta = 1 - \frac{\delta\left(1+\beta\left|\gamma\right|\right)\Gamma(\vartheta+1)Q(\vartheta+2)}{2\left[\left(1+\beta\left|\gamma\right|\right)\Gamma\left(\vartheta+1\right)Q\left(\vartheta+2\right)+4\beta\left|\gamma\right|\Gamma\left(\vartheta+2\right)Q(\vartheta)\right]} \tag{22}$$

then $\mathcal{N}_{n,\delta}\left(g\right) \subset \mathcal{M}_{\vartheta}^{n}(\beta,\gamma)$.

Proof. Let $f(z) \in \mathcal{N}_{n,\delta}(g)$. Then,

$$\sum_{n=2}^{\infty} n\,|a_n - b_n| \leq \delta, \tag{23}$$

which yields the coefficient inequality,

$$\sum_{n=2}^{\infty} |a_n - b_n| \leq \frac{\delta}{2} \qquad (n \in \mathbb{N}).$$

Since $g(z) \in \mathcal{M}_{\vartheta}^{n}(\beta,\gamma)$ by (18), we have

$$\sum_{n=2}^{\infty} b_n \leq \frac{-4\beta\left|\gamma\right|\Gamma(\vartheta+2)\,Q(\vartheta)}{(1+\beta\left|\gamma\right|)\Gamma(\vartheta+1)Q(\vartheta+2)}, \tag{24}$$

and so

$$\begin{aligned}\left|\frac{f(z)}{g(z)}-1\right| &< \frac{\sum\limits_{n=2}^{\infty}|a_n-b_n|}{1-\sum\limits_{n=2}^{\infty}b_n}\\ &\leq \frac{\delta}{2}\,\frac{\frac{\Gamma(\vartheta+1)Q(\vartheta+2)}{4\Gamma(\vartheta+2)Q(\vartheta)}(1+\beta\left|\gamma\right|)}{\frac{\Gamma(\vartheta+1)Q(\vartheta+2)}{4\Gamma(\vartheta+2)Q(\vartheta)}(1+\beta\left|\gamma\right|)+\beta\left|\gamma\right|}\\ &= 1-\eta.\end{aligned}$$

Thus, by definition, $f(z) \in \mathcal{M}_{\vartheta}^{n}(\beta,\gamma)$ for η given by (22), which establishes the desired result. □

Theorem 4.3. *If* $g(z) \in \mathcal{R}_{\vartheta}^{n}(\beta,\gamma,\mu)$ *and*

$$\eta = 1 - \frac{\delta\,(1+\mu)\,\Gamma(\vartheta+1)Q(\vartheta+2)}{2\left[(1+\mu)\,\Gamma(\vartheta+1)\,Q(\vartheta+2)+4\beta\left|\gamma\right|\Gamma(\vartheta+2)\,Q(\vartheta)\right]} \tag{25}$$

then $\mathcal{N}_{n,\delta}(g) \subset \mathcal{R}_{\vartheta}^{n}(\beta,\gamma,\mu)$.

Proof. We omit the proofs since it is similar to Theorem 4.2. □

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