TURKISH JOURNAL OF SCIENCE
VOLUME 6, ISSUE 1, 24–30
ISSN: 2587–0971

http://dergipark.gov.tr/tjos

Analytic Functions Expressed with q-Poisson Distribution Series

Nizami Mustafa[a], Veysel Nezir[b]

[a] *Kafkas University, Faculty of Science and Letters, Department of Mathematics, Kars, Turkey*
[b] *Kafkas University, Faculty of Science and Letters, Department of Mathematics, Kars, Turkey*

Abstract. Recently, the q- derivative operator has been used to investigate several subclasses of analytic functions in different ways with different perspectives by many researchers and their interesting results are too voluminous to discuss. The q- derivative operator are also used to construct some subclasses of analytic functions.

In this study, we introduce certain subclasses of analytic and univalent functions in the open unit disk defined by q-derivative. Here, we give some conditions for an analytic and univalent function to belonging to these classes. Also, in the study, we define two functions using q-derivative and we aim to find the conditions for this functions to belonging to defined above subclasses of analytic functions.

1. Intoduction

Let A be the class of analytic functions f in the open unit disk $U = \{z \in C : |z| < 1\}$, normalized by $f(0) = 0 = f'(0) - 1$ of the form

$$f(z) = z + a_2z^2 + a_3z^3 + \cdots + a_nz^n + \cdots = z + \sum_{n=2}^{\infty} a_nz^n,\ a_n \in C. \tag{1}$$

Also, by Swe will denote the family of all functions in A which are univalent in U.

Let T denote the subclass of all functions f in A of the form

$$f(z) = z - a_2z^2 - a_3z^3 - \cdots - a_nz^n - \cdots = z - \sum_{n=2}^{\infty} a_nz^n,\ a_n \geq 0. \tag{2}$$

Some of the important and well-investigated subclasses of the univalent functions class S include the classes $S^*(\alpha)$ and $C(\alpha)$, respectively, starlike and convex functions of order $\alpha(\alpha \in [0,1))$. By definition, we have (see for details, [2, 3], also [9])

$$S^*(\alpha) = \left\{ f \in A : \ \mathrm{Re}\left(\frac{zf'(z)}{f(z)}\right) > \alpha,\ z \in U \right\},\ C(\alpha) = \left\{ f \in A : \ \mathrm{Re}\left(1 + \frac{zf''(z)}{f'(z)}\right) > \alpha,\ z \in U \right\}.$$

Corresponding author: VN mail address: veyselnezir@yahoo.com ORCID:0000-0001-9640-8526, NM ORCID:0000-0002-2758-0274
Received: 15 February 2021; Accepted: 3 April 2021; Published: 30 April 2021
Keywords. q-poisson distribution, analytic function, univalent function, q-derivative, starlike function, convex function
2010 *Mathematics Subject Classification*. 30C45, 30C50, 30C55, 30C80
Cited this article as: Nezir V, Mustafa N. Analytic Functions Expressed with q-Poisson Distribution Series, Turkish Journal of Science. 2021, 6(1), 24-30.

For $\beta \in [0,1)$, interesting generalization of the classes $S^*(\alpha)$ and $C(\alpha)$ are the classes $S^*(\alpha,\beta)$ and $C(\alpha,\beta)$ which, respectively, defined as follows

$$S^*(\alpha,\beta) = \left\{ f \in S : \ \mathrm{Re}\left(\frac{zf'(z)}{\beta z f'(z) + (1-\beta) f(z)} \right) > \alpha, \ z \in U \right\},$$

$$C(\alpha,\beta) = \left\{ f \in S : \ \mathrm{Re}\left(\frac{f'(z) + zf''(z)}{f'(z) + \beta z f''(z)} \right) > \alpha, \ z \in U \right\}.$$

The classes $TS^*(\alpha,\beta)$ and $TC(\alpha,\beta)$ were extensively studied by Altintaş and Owa [1] and certain conditions for hypergeometric functions and generalized Bessel functions for these classes were studied by Moustafa [5] and Porwal and Dixit [8].

For $\gamma \in [0,1]$, a generalization of the function classes $S^*(\alpha,\beta)$ and $C(\alpha,\beta)$ is the class $S^*C(\alpha,\beta;\gamma)$ which is defined as follows:

$$S^*C(\alpha,\beta;\gamma) = \left\{ f \in S : Re\left(\frac{zf'(z) + \gamma z^2 f''(z)}{\gamma z (f'(z) + \beta z f''(z)) + (1-\gamma)(\beta z f'(z) + (1-\beta) f(z))} \right) > \alpha \right\}, z \in U.$$

In his fundamental paper [4], Jackson, for $q \in (0,1)$, introduced the q-derivative operator D_q of the an analytic function f as follows:

$$D_q f(z) = \begin{cases} \frac{f(z)-f(qz)}{(1-q)z}, & if \ \ z \neq 0, \\ f'(0) & , if \ \ z = 0. \end{cases}$$

The formulas for the q- derivative D_q of a product and a quotient of functions are

$$D_q z^n = [n]_q z^{n-1}, n \in \mathrm{N},$$

where

$$[n]_q = \frac{1-q^n}{1-q} = \sum_{k=1}^{n} q^{k-1}$$

is the q- analogue of the natural number n.

It is clear that $\lim_{q \to 1^-} [n]_q = n$, $[0]_q = 0$ and $\lim_{q \to 1^-} D_q f(z) = f'(z)$ for the function $f \in A$.

For $q \in (0,1)$ and $\alpha \in [0,1)$, we define by $S_q^*(\alpha)$ and $C_q(\alpha)$ the subclass of A which we will call, respectively, q - starlike and q- convex functions of order α, as follows:

$$S_q^*(\alpha) = \left\{ f \in S : \ Re \frac{z D_q f(z)}{f(z)} > \alpha, z \in U \right\}, C_q(\alpha) = \left\{ f \in S : \ Re \frac{D_q (z D_q f(z))}{D_q f(z)} > \alpha, z \in U \right\}.$$

Also, let's denote $TS_q^*(\alpha) = T \bigcap S_q^*(\alpha)$ and $TC_q(\alpha) = T \bigcap C_q(\alpha)$.

For $\beta \in [0,1)$, interesting generalization of the function classes $S_q^*(\alpha)$ and $C_q(\alpha)$ are the function classes $S_q^*(\alpha,\beta)$ and $C_q(\alpha,\beta)$, respectively, which we define as follows:

$$S_q^*(\alpha,\beta) = \left\{ f \in A : \ \mathrm{Re}\left(\frac{z D_q f(z)}{\beta z D_q f(z) + (1-\beta) f(z)} \right) > \alpha, \ z \in U \right\}, C_q(\alpha,\beta) = \left\{ f \in A : \ \mathrm{Re}\left(\frac{D_q f(z) + z D_q^2 f(z)}{D_q f(z) + \beta z D_q^2 f(z)} \right) > \alpha, \ z \in U \right\}.$$

Now let's define a generalization of the function classes $S_q^*(\alpha,\beta)$ and $C_q(\alpha,\beta)$ as follows:

Definition 1.1. *For $\alpha, \beta \in [0,1)$ and $\gamma \in [0,1]$ a function f given by* (1) *is said to be in the class $S_q^*C_q(\alpha,\beta;\gamma)$ if the following condition is satisfied*

$$Re\left(\frac{zD_qf(z) + \gamma z^2 D_q^2 f(z)}{\gamma z\left(D_qf(z) + \beta z D_q^2 f(z)\right) + (1-\gamma)\left(\beta z D_q f(z) + (1-\beta) f(z)\right)}\right) > \alpha, z \in U.$$

We will use $TS_q^*C_q(\alpha,\beta;\gamma) = T \cap S_q^*C_q(\alpha,\beta;\gamma)$.

It is clear that $S_q^*C_q(\alpha,\beta;0) = S_q^*(\alpha,\beta)$, $S_q^*C_q(\alpha,\beta;1) = C_q(\alpha,\beta)$, $\lim_{q\to 1^-} S_q^*C_q(\alpha,\beta;\gamma) = S^*C(\alpha,\beta;\gamma)$ and $\lim_{q\to 1^-} TS_q^*C_q(\alpha,\beta;\gamma) = TS^*C(\alpha,\beta;\gamma)$. So, function classes $S_q^*C_q(\alpha,\beta;\gamma)$ and $TS_q^*C_q(\alpha,\beta;\gamma)$ are generalization of the previously known function classes $S_q^*(\alpha,\beta)$, $C_q(\alpha,\beta)$, $S^*C(\alpha,\beta;\gamma)$ and $TS^*C(\alpha,\beta;\gamma)$ of analytic functions, respectively.

A variable x is said to have q- Poisson Distribution if it takes the values 0,1,2,3,... with probabilities e_q^{-p}, $\frac{p}{1!}e_q^{-p}$, $\frac{p^2}{2!}e_q^{-p}$, $\frac{p^3}{3!}e_q^{-p}$, ... , respectively, where p a parameter and

$$e_q^x = 1 + \frac{x}{[1]_q!} + \frac{x^2}{[2]_q!} + \frac{x^3}{[3]_q!} + \cdots + \frac{x^n}{[n]_q!} + \cdots = \sum_{n=0}^{\infty} \frac{x^n}{[n]_q!} \tag{3}$$

is q- analogue of the exponential function e^x and

$$[n]_q! = [1]_q \cdot [2]_q \cdot [3]_q \cdots [n]_q$$

is the q- analogue of the factorial function $n! = 1 \cdot 2 \cdot 3 \cdots n$.

Thus, for q- Poisson Distribution, we have

$$P_q(x = n) = \frac{p^n}{n!} e_q^{-p},\ n = 0, 1, 2, 3, ...$$

Now, we introduce a q- Poisson Distribution series as follows:

$$z + \sum_{n=2}^{\infty} \frac{p^{n-1} e_q^{-p}}{[n-1]_q!} z^n,\ z \in U. \tag{4}$$

We can easily show that series (4) is convergent and the radius of convergence is infinity.

Let us define functions $F_q : \mathrm{C} \to \mathrm{C}$ and $G_q : \mathrm{C} \to \mathrm{C}$ as

$$F_q(z) = z + \sum_{n=2}^{\infty} \frac{p^{n-1} e_q^{-p}}{[n-1]_q!} z^n,\ z \in U \tag{5}$$

and

$$G_q(z) = 2z - F_q(z) = z - \sum_{n=2}^{\infty} \frac{p^{n-1} e_q^{-p}}{[n-1]_q!} z^n,\ z \in U. \tag{6}$$

It is clear that $F_q \in A$ and $G_q \in T$, respectively.

In this study, using $q-$derivative we introduce certain subclasses of analytic and univalent functions in the open unit disk in the complex plane. Here, we give some conditions for an analytic and univalent function to belong to these classes. Applications of a q- Poisson Distribution series on the analytic functions are also given. In the study, we define two functions F_q and G_q by q- Poisson Distribution and we aim to find the conditions for this functions to belong to the classes of analytic functions which we define in the study.

2. Main Results

In this section, we will give sufficient condition for the function F_q defined by (5), belonging to the class $S_q^*C_q(\alpha,\beta;\gamma)$, and necessary and sufficient condition for the function G_q defined by (6), belonging to the class $TS_q^*C_q(\alpha,\beta;\gamma)$, respectively.

In order to prove our main results, we need the following theorems.

Theorem 2.1. *[6] Let $f\in A$. Then, $f\in S_q^*C_q(\alpha,\beta;\gamma)$ if the following condition is satisfied*

$$\sum_{n=2}^{\infty}\left\{[n]_q\left[(1-\alpha\beta)\left(1+\gamma[n-1]_q\right)-(1-\beta)\alpha\gamma\right]-\alpha(1-\beta)(1-\gamma)\right\}|a_n|\le 1-\alpha.$$

The result obtained here is sharp.

Theorem 2.2. *[6] Let $f\in T$. Then, $f\in TS_q^*C_q(\alpha,\beta;\gamma)$ if and only if*

$$\sum_{n=2}^{\infty}\left\{[n]_q\left[(1-\alpha\beta)\left(1+\gamma[n-1]_q\right)-(1-\beta)\alpha\gamma\right]-\alpha(1-\beta)(1-\gamma)\right\}|a_n|\le 1-\alpha.$$

The result obtained here is sharp.

A sufficient condition for the function F_q defined by (5) to belonging to the class $S_q^*C_q(\alpha,\beta;\gamma)$ is given by the following theorem.

Theorem 2.3. *Let $p>0$ and the following condition is provided*

$$\left\{\begin{array}{l}(1-\alpha\beta)\gamma p^2+\left\{(1-\alpha\beta)\left[1+(1+q)q\gamma e_q^{p(q-1)}\right]-(1-\beta)\alpha\gamma\right\}p\\ -[1-\alpha\beta-(1-\beta)\alpha\gamma]\left(1-e_q^{p(q-1)}\right)\end{array}\right\}e_q^p\le 1-\alpha. \tag{7}$$

Then, the function F_q defined by (5) *belongs to the class$S_q^*C_q(\alpha,\beta;\gamma)$.*

Proof. Since $F_q\in A$ and

$$F_q(p,z)=z+\sum_{n=2}^{\infty}\frac{p^{n-1}}{[n-1]!}e_q^{-p}z^n,\ z\in U,$$

according to Theorem 2.1, we must show that

$$\sum_{n=2}^{\infty}\left\{[n]_q\left[(1-\alpha\beta)\left(1+\gamma[n-1]_q\right)-(1-\beta)\alpha\gamma\right]-\alpha(1-\beta)(1-\gamma)\right\}\frac{p^{n-1}}{[n-1]_q!}e_q^{-p}\le 1-\alpha. \tag{8}$$

Let

$$L_q(\alpha,\beta,\gamma)=\sum_{n=2}^{\infty}\left\{[n]_q\left[(1-\alpha\beta)\left(1+\gamma[n-1]_q\right)-(1-\beta)\alpha\gamma\right]-\alpha(1-\beta)(1-\gamma)\right\}\frac{p^{n-1}}{[n-1]_q!}e_q^{-p}.$$

By setting

$$\begin{aligned}&[n]_q\left[(1-\alpha\beta)\left(1+\gamma[n-1]_q\right)-(1-\beta)\alpha\gamma\right]-\alpha(1-\beta)(1-\gamma)\\&=[n]_q[1-\alpha\beta-(1-\beta)\alpha\gamma]+[n]_q[n-1]_q(1-\alpha\beta)\gamma-\alpha(1-\beta)(1-\gamma)\end{aligned}$$

and using $[n]_q=[n-1]_q+q^{n-1}$, $[n]_q=[n-2]_q+q^{n-2}+q^{n-1}$, we write

$$\begin{aligned}&[n]_q\left[(1-\alpha\beta)\left(1+\gamma[n-1]_q\right)-(1-\beta)\alpha\gamma\right]-\alpha(1-\beta)(1-\gamma)\\&=(1-\alpha\beta)\gamma[n-2]_q[n-1]_q+\left[1-\alpha\beta-(1-\beta)\alpha\gamma+(1-\alpha\beta)\gamma\left(q^{n-2}+q^{n-1}\right)\right][n-1]_q\\&+q^{n-1}[1-\alpha\beta-(1-\beta)\alpha\gamma]-\alpha(1-\beta)(1-\gamma)\end{aligned} \tag{9}$$

Considering equality (9), by simple computation, we can write

$$L_q(\alpha,\beta,\gamma;p) = (1-\alpha\beta)\,\gamma e_q^{-p}\sum_{n=3}^{\infty}\frac{p^{n-1}}{[n-3]_q!} + [1-\alpha\beta-(1-\beta)\,\alpha\gamma]\,e_q^{-p}\sum_{n=2}^{\infty}\frac{p^{n-1}}{[n-2]_q!}$$
$$+(1-\alpha\beta)(1+q)\,\gamma e_q^{-p}\sum_{n=2}^{\infty}\frac{(qp)^{n-1}}{[n-2]_q!} + [1-\alpha\beta-(1-\beta)\,\alpha\gamma]\,e_q^{-p}\sum_{n=2}^{\infty}\frac{(qp)^{n-1}}{[n-1]_q!}$$
$$-\alpha(1-\beta)(1-\gamma)\,e_q^{-p}\sum_{n=2}^{\infty}\frac{p^{n-1}}{[n-1]_q!}.$$

Then, using the equality (3), we obtain

$$L_q(\alpha,\beta,\gamma;p) = (1-\alpha\beta)\,\gamma p^2 + \left\{(1-\alpha\beta)\left[1+(1+q)\,q\gamma e_q^{p(q-1)}\right] - (1-\beta)\,\alpha\gamma\right\}p$$
$$-[1-\alpha\beta-(1-\beta)\,\alpha\gamma]\left(1-e_q^{p(q-1)}\right) + (1-\alpha)\left(1-e_q^{-p}\right).$$

Therefore, inequality (8) holds true if

$$(1-\alpha\beta)\,\gamma p^2 + \left[1-\alpha\beta-(1-\beta)\,\alpha\gamma+(1-\alpha\beta)(1+q)\,q\gamma e_q^{p(q-1)}\right]p$$
$$-[1-\alpha\beta-(1-\beta)\,\alpha\gamma]\left(1-e_q^{p(q-1)}\right) + (1-\alpha)\left(1-e_q^{-p}\right) \le 1-\alpha$$

which is equivalent to (7).

Thus, the proof of Theorem 2.3 is completed.

□

From the Theorem 2.3, we can readily deduce the following results.

Corollary 2.4. *If* $p > 0$ *and satisfied the following condition*

$$(1-\alpha\beta)\left(p-1+e_q^{p(q-1)}\right)e_q^{p} \le 1-\alpha,$$

then the function F_q *defined by* (5) *belongs to the class* $S_q^*(\alpha,\beta)$.

Corollary 2.5. *If* $p > 0$ *and satisfied the following condition*

$$\left\{(1-\alpha\beta)\,p^2 + \left[1-\alpha+(1-\alpha\beta)(1+q)\,qe_q^{p(q-1)}\right]p - (1-\alpha)\left(1-e_q^{p(q-1)}\right)\right\}e_q^{p} \le 1-\alpha,$$

then the function F_q *defined by* (5) *belongs to the class* $C_q(\alpha,\beta)$.

Now, we give necessary and sufficient condition for the function G_q defined by (6), to belonging to the class $TS_q^*C_q(\alpha,\beta;\gamma)$ with the following theorem.

Theorem 2.6. *If* $p > 0$, *then the function* G_q *defined by* (6) *belongs to the class* $TS_q^*C_q(\alpha,\beta;\gamma)$ *if and only if satisfied the following condition*

$$\left\{\begin{array}{l}(1-\alpha\beta)\,\gamma p^2 + \left\{(1-\alpha\beta)\left[1+(1+q)\,q\gamma e_q^{p(q-1)}\right]-(1-\beta)\,\alpha\gamma\right\}p\\ -[1-\alpha\beta-(1-\beta)\,\alpha\gamma]\left(1-e_q^{p(q-1)}\right)\end{array}\right\}e_q^{p} \le 1-\alpha. \qquad (10)$$

Proof. Firstly, let us prove the sufficiency of the theorem.

First of all, let us state that we will use Theorem 2.2 to prove the theorem.

It is clear that $G_q \in T$. Let us show that the function G_q satisfies the sufficiency condition of Theorem 2.2

.

From the proof of Theorem 2.3, we write

$$\begin{aligned}&\textstyle\sum_{n=2}^{\infty}\left\{[n]_q\left[(1-\alpha\beta)\left(1+\gamma[n-1]_q\right)-(1-\beta)\alpha\gamma\right]-\alpha(1-\beta)(1-\gamma)\right\}\frac{p^{n-1}}{[n-1]_q!}e_q^{-p}\\ &=(1-\alpha\beta)\gamma p^2+\left\{(1-\alpha\beta)\left[1+(1+q)q\gamma e_q^{p(q-1)}\right]-(1-\beta)\alpha\gamma\right\}p\\ &-[1-\alpha\beta-(1-\beta)\alpha\gamma]\left(1-e_q^{p(q-1)}\right)+(1-\alpha)\left(1-e_q^{-p}\right)\end{aligned} \tag{11}$$

Now, suppose that condition (10) is satisfied.
It follows that

$$\begin{aligned}&(1-\alpha\beta)\gamma p^2+\left\{(1-\alpha\beta)\left[1+(1+q)q\gamma e_q^{p(q-1)}\right]-(1-\beta)\alpha\gamma\right\}p-[1-\alpha\beta-(1-\beta)\alpha\gamma]\left(1-e_q^{p(q-1)}\right)\\ &\leq(1-\alpha)e_q^{-p},\end{aligned}$$

which is equivalent to

$$\begin{aligned}&(1-\alpha\beta)\gamma p^2+\left\{(1-\alpha\beta)\left[1+(1+q)q\gamma e_q^{p(q-1)}\right]-(1-\beta)\alpha\gamma\right\}p-[1-\alpha\beta-(1-\beta)\alpha\gamma]\left(1-e_q^{p(q-1)}\right)\\ &+(1-\alpha)\left(1-e_q^{-p}\right)\leq 1-\alpha,\end{aligned}$$

Considering equality (11), we write

$$\sum_{n=2}^{\infty}\left\{[n]_q\left[(1-\alpha\beta)\left(1+\gamma[n-1]_q\right)-(1-\beta)\alpha\gamma\right]-\alpha(1-\beta)(1-\gamma)\right\}\frac{p^{n-1}}{[n-1]_q!}e_q^{-p}\leq 1-\alpha. \tag{12}$$

Thus, the function G_q satisfies the sufficiency condition of Theorem 2.2. Hence, according to Theorem 2.2, the function G_q belongs to the class $TS_q^*C_q(\alpha,\beta;\gamma)$.

With this, the proof of the sufficiency of theorem is completed.

Now, let us we prove the necessity of theorem.

Assume that $G_q \in TS_q^*C_q(\alpha,\beta;\gamma)$. Then, from Theorem 2, we can say that condition (12) is satisfied.

It follows from (11) that

$$\begin{aligned}&(1-\alpha\beta)\gamma p^2+\left\{(1-\alpha\beta)\left[1+(1+q)q\gamma e_q^{p(q-1)}\right]-(1-\beta)\alpha\gamma\right\}p-[1-\alpha\beta-(1-\beta)\alpha\gamma]\left(1-e_q^{p(q-1)}\right)\\ &+(1-\alpha)\left(1-e_q^{-p}\right)\leq 1-\alpha,\end{aligned}$$

which is equivalent to (10).

This completes proof of the necessity of theorem.

Thus, the proof of Theorem 2.6 is completed.

□

From Theorem 2.6, we can readily deduce the following results.

Corollary 2.7. *If $p>0$, then the function G_q defined by* (6) *belongs to the class $TS_q^*(\alpha,\beta)$ if and only if satisfied the following condition*

$$(1-\alpha\beta)\left(p-1+e_q^{p(q-1)}\right)e_q^{p}\leq 1-\alpha.$$

Corollary 2.8. *If $p>0$, then the function G_q defined by* (6) *belongs to the class $TC_q(\alpha,\beta)$ if and only if satisfied the following condition*

$$\left\{(1-\alpha\beta)p^2+\left[1-\alpha+(1-\alpha\beta)(1+q)qe_q^{p(q-1)}\right]p-(1-\alpha)\left(1-e_q^{p(q-1)}\right)\right\}e_q^{p}\leq 1-\alpha.$$

Remark 2.9. *The results obtained in Theorem 2.3, Theorem 2.6 and Corollaries 2.4, 2.5, 2.7, 2.8 are generalization of the results obtained in Theorem 3,4 and Corollary 5-8 in [7].*

References

[1] Altintaş O, Owa S. On subclasses of univalent functions with negative coefficients. Pusan Kyongnam Mathematical Journal. 4, 1988, 41–56.
[2] Duren PL. Univalent Functions. Grundlehren der Mathematischen Wissenshaften, Bd. 259, New York, Springer-Verlag, Tokyo, 1983, 382p.
[3] Goodman AW. Univalent Functions. Volume I, Polygonal, Washington, 1983, 246p.
[4] Jackson FH. On q-functions and a certain difference operatör. Trans. Roy. Soc. Edin. 46, 1908, 253–281.
[5] Moustafa AO. A study on starlike and convex properties for hypergeometric functions. Journal of Inequalities in Pure and Applied Mathematics. 10(3), 2009, article 87, 1–16.
[6] Mustafa N, Nezir V. On Subclasses of Analytic Functions Defined by q- Derivative and their Some Geometric Properties. 3^{rd} International Conference on Mathematical and Related Sciences: Current Trends and Developments, ICMRS – 2020, 20-22 November, 2020, pp 129–1335.
[7] Mustafa N, Korkmaz S. Analytic Functions Expressed with Poisson Distribution Series and their Some Properties. Journal of Contemporary Applied Mathematics, 2020 (submitted).
[8] Porwal S, Dixit KK. An application of generalized Bessel functions on certain analytic functions. Acta Universitatis Matthiae Belii. Series Mathematics, 2013, 51–57.
[9] Srivastava HM, Owa S. Current Topics in Analytic Function Theory. World Scientific, Singapore, 1992, 456p.