TURKISH JOURNAL OF SCIENCE
VOLUME 6, ISSUE 2, 89–95
ISSN: 2587–0971

http://dergipark.gov.tr/tjos

An examination on $N - D^*$ partner curves with common principal normal and Darboux vector in $\mathbf{E}^3$.

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Abstract. In this paper, we define and examine curves with common principal normal and Darboux vectors such that the principal normal vector of first curve and the Darboux vector of second curve are linearly dependent. We name the first curve as $N - D^*$ curve, and the second curve as the $N - D^*$ partner curve. These new curves are called $N - D^*$ pair curves. Also we give the Frenet-Serret apparatus of the second curve based on the Frenet-Serret apparatus of first curve.

1. Introduction and Preliminaries

The evolute and involute curve are the curves whose tangent lines intersect orthogonally, hence the principal normal vector of the first curve and tangent vector of second curve are linearly dependent. So if that is the case, then first curve is named to be evolute, and the second curve is called as involute curve. For more detail see in [2], [3].

Mannheim curve examined first by A. Mannheim in 1878 is a curve if and only if $\kappa/\left(\kappa^2+\tau^2\right)$ is a nonzero constant, where κ is the curvature and τ is the torsion. Also, a new definition of these associated curves was given by Liu and Wang (2008); if the principal normal vector of the first curve and binormal vector of the second curve are linearly dependent, then the first curve is called Mannheim curve, and the second curve is called Mannheim partner curve. As a result they called these new curves as Mannheim pair. For more detail see in [4]. Bertrand pair curves are another special curves with common principal normal lines. A curve is Bertrand curve, if and only if there exist nonzero real constant numbers λ and β such that $\lambda\kappa + \beta\tau = 1$. For more detail see in [5]. Before in [6, 7], we produced some other new partner curves by using similar way.

By this study, it is of interest to us to define a new curve pair such that there exist a linear dependence between the principal normal and the Darboux vectors. By doing so, we introduce a new concept such that $N - D^*$ partner curves and examine some of their invariants.

2. $N - D^*$ pair curves

Let α and α^* be the curves with Frenet-Serret apparatus $\{T, N, B, D, \kappa, \tau\}$ and $\{T^*, N^*, B^*, D^*, \kappa^*, \tau^*\}$, where $\kappa, \kappa^*$and τ, τ^* are the curvature functions of the first and the second curve, respectively, and $D = \dfrac{\tau T + \kappa B}{\sqrt{\kappa^2+\tau^2}}$ is

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Received: 12 July 2021; Accepted: 19 August 2021; Published: 30 September 2021
Keywords. Darboux vector, Offset curves, Mannheim curves, Bertrand pairs
2010 *Mathematics Subject Classification*. 14H45, 53A04.
Cited this article as: Kılıçoğlu Ş, Şenyurt S. An examination on $N - D^*$ partner curves with common principal normal and Darboux vector in $\mathbf{E}^3$., Turkish Journal of Science. 2021, 6(2), 89-95.

unit Darbux vector field discovered by Gaston Darboux as the areal velocity vector of the Frenet frame of a space curve. The Darboux vector field of any arclengthed curve α has symmetrical properties [1]: $D\times T = T'$; $D\times N = N'$; $D\times B = B'$. Similarly, $D^* = \dfrac{\tau^* T^* + \kappa^* B^*}{\sqrt{\kappa^{*2} + \tau^{*2}}}$ as the unit Darboux vector field of second curve α^*, in E^3.

Definition 2.1. *If the principal normal vector of first curve and unit Darboux vector D^* of second curve are linearly dependent, then first curve is called $N - D^*$ curve, and the second curve is called $N - D^*$partner curve. As a result we called these new curves as $N - D^*$ pair curves with the following equations:*

$$\alpha^*(s) = \alpha(s) + \lambda(s)D^*(s),$$

and under the condition $N = D^*$

$$\alpha^*(s) = \alpha(s) + \lambda(s)N(s).$$

Corollary 2.2. *The distance between $N - D^*$ pair curves is $|\lambda|$.*

2.1. Tangent vector field of $N - D^*$ partner curve

Theorem 2.3. *If the tangent vector field of $N - D^*$ partner curve is T^*, then it can be given based on the Frenet apparatus of the first curve as*

$$T^* = \left(\frac{1-\lambda\kappa}{\lambda'}T + N + \frac{\lambda\tau}{\lambda'}B\right)\cos\theta,$$

where $\dfrac{ds}{ds^*} = \dfrac{1}{\sqrt{\delta}}$, and $\delta = (1-\kappa\lambda)^2 + \lambda'^2 + \tau^2\lambda^2$. Also $\dfrac{\lambda'}{\sqrt{\delta}} = \cos\theta$, $\theta = \sphericalangle(T^*, D^*)$, $0 < \theta < \pi/2$.

Proof. Since $\alpha^* = \alpha + \lambda N$, and taking its derivative with respect to it's arclength parameter s^* we have

$$\begin{aligned}\frac{d\alpha^*}{ds^*} &= \frac{d(\alpha + \lambda N)}{ds}\frac{ds}{ds^*},\\ &= \left((1-\kappa\lambda)T + \lambda' N + \lambda\tau B\right)\frac{ds}{ds^*}.\end{aligned}$$

and $\left\|\dfrac{d\alpha^*}{ds}\right\| = \sqrt{(1-\lambda\kappa)^2 + \lambda'^2 + \lambda^2\tau^2} = \sqrt{\delta}$. Also α^* is an arc-lengthed curve with the s^*; $\left\langle\dfrac{d\alpha^*}{ds^*}, \dfrac{d\alpha^*}{ds^*}\right\rangle = 1$, hence

$$\frac{ds}{ds^*} = \frac{1}{\sqrt{(1-\kappa\lambda)^2 + \lambda'^2 + (\lambda\tau)^2}} = \frac{1}{\sqrt{\delta}}.$$

Now, we can write the tangent vector field as

$$T^* = \frac{(1-\kappa\lambda)T + \lambda' N + \tau\lambda B}{\sqrt{\delta}}.$$

Let $\sphericalangle(T^*, D^*) = \theta$, $0 < \theta < \pi/2$, so

$$\langle T^*, N\rangle = \langle D^*, T^*\rangle = \|T^*\|\,\|D^*\| = \cos\theta.$$

Since

$$\langle T^*, N\rangle = \left\langle\frac{(1-\kappa\lambda)T + \lambda' N + \tau\lambda B}{\sqrt{(1-\kappa\lambda)^2 + \lambda'^2 + \tau^2\lambda^2}}, N\right\rangle = \frac{\lambda'}{\sqrt{\delta}},$$

we have

$$\left\langle T^*, \tilde{D}^*\right\rangle = \|T^*\|\,\|D^*\|\cos\theta = \cos\theta.$$

So there is the relationship among the curvatures and θ as in the following way

$$\frac{\lambda'}{\cos\theta} = \sqrt{\delta}.$$

By utilizing this latter relation, we have the proof as in the following

$$T^* = \left(\frac{1-\lambda\kappa}{\lambda'}T + N + \frac{\lambda\tau}{\lambda'}B\right)\cos\theta.$$

□

Theorem 2.4. *There is the relationship among the λ, curvatures of $N-D^*$ curve and θ , based on the Frenet-Serret apparatus as in the following way*

$$\tan\theta = \frac{\sqrt{(1-\kappa\lambda)^2+\tau^2\lambda^2}}{\lambda'}.$$

Proof. Since $\delta = \dfrac{\lambda'^2}{\cos^2\theta}$ and $\delta = (1-\kappa\lambda)^2 + \lambda'^2 + \tau^2\lambda^2$, it is trivial

$$(1-\kappa\lambda)^2 + \lambda'^2 + \tau^2\lambda^2 = \frac{\lambda'^2}{\cos^2\theta},$$

$$\lambda'^2\left(1-\sec^2\theta\right) + (1-\kappa\lambda)^2 + \tau^2\lambda^2 = 0,$$

$$\tan^2\theta = \frac{(1-\kappa\lambda)^2+\tau^2\lambda^2}{\lambda'^2}.$$

□

Theorem 2.5. *There is the relationship among the curvatures of $N-D^*$ curve λ, and angle θ based on the Frenet-Serret apparatus as in the following way*

$$\delta' = \frac{2\left(\lambda'' - \left(\kappa^2+\tau^2\right)\lambda + \kappa\right)}{\cos\theta}.$$

Proof. Since $\langle N^*, T^*\rangle = 0$, $\langle N^*, B^*\rangle = 0$, the principal normal vector N^* of the second $N-D^*$ partner curve is perpendicular to its Darboux vector; $\langle N^*, D^*\rangle = 0$, Note that we also have $N = D^*$. Hence for the principal normal vector fields N we have that $\langle N, N^*\rangle = 0$. Since $\dfrac{dT^*}{ds^*} = \kappa^* N^*$ we have

$$\frac{dT^*}{ds} = \frac{1}{\delta}\left([(1-\kappa\lambda)T + \lambda' N + \tau\lambda B]'\ \sqrt{\delta} - [(1-\kappa\lambda)T + \lambda' N + \tau\lambda B]\ \sqrt{\delta'}\right).$$

Since $\dfrac{1}{\kappa^*} \neq 0$, $\dfrac{ds}{ds^*} \neq 0$, $\dfrac{1}{\delta} \neq 0$, then $\left\langle N, \dfrac{dT^*}{ds}\right\rangle = 0$, hence

$$\left\langle N, \frac{((1-\kappa\lambda)T + \lambda' N + \tau\lambda B')'\ \sqrt{\delta}}{\delta}\right\rangle - \left\langle N, \frac{((1-\kappa\lambda)T + \lambda' N + \tau\lambda B')}{\delta}\ \sqrt{\delta'}\right\rangle = 0,$$

$$\left\langle N, (1-\kappa\lambda)' T + (1-\kappa\lambda)T' + \lambda'' N + \lambda' N' + (\tau\lambda)' B + \tau\lambda B'\right\rangle - \lambda'\ \sqrt{\delta'} = 0.$$

$$\left\langle N, (1-\kappa\lambda)\,\kappa N + \lambda'' N - \lambda\tau^2 N\right\rangle - \lambda'\sqrt{\delta}' = 0,$$

$$\lambda'' - \left(\kappa^2+\tau^2\right)\lambda + \kappa = \frac{\cos\theta}{2}\delta',$$

$$\frac{\lambda'}{\sqrt{\delta}} = \cos\theta,$$

$$\frac{2\left(\lambda'' - \left(\kappa^2+\tau^2\right)\lambda + \kappa\right)}{\lambda'} = \frac{\delta'}{\sqrt{\delta}}.$$

□

2.2. *First curvature of N − D* partner curve*

Theorem 2.6. *If the first curvature of $N - D^*$ partner curve is κ^*, then it can be given based on the Frenet apparatus of the first curve as in the following way*

$$\kappa^* = \frac{\cos\theta}{2\lambda'}\left(\begin{array}{c} \left((\kappa'\lambda + 2\lambda'\kappa) + (1-\kappa\lambda)\left(\frac{\lambda'}{\cos\theta}\right)'\right)^2 + \left(\left(\kappa - \lambda\left(\kappa^2+\tau^2\right)+\lambda''\right) - \lambda'\left(\frac{\lambda'}{\cos\theta}\right)'\right)^2 \\ + \left((2\tau'\lambda + \tau\lambda') - \lambda\tau\left(\frac{\lambda'}{\cos\theta}\right)'\right)^2 \end{array}\right)^{\frac{1}{2}}$$

Proof. Since $\kappa^* N^* = \dfrac{dT^*}{ds}\dfrac{ds}{ds^*}$ and $\dfrac{ds}{ds^*} = \dfrac{1}{\sqrt{\delta}}$ it can be calculated as

$$\kappa^* N^* = \frac{1}{\sqrt{\delta^3}}\left[\begin{array}{c} \left[((1-\kappa\lambda)' - \lambda'\kappa)\sqrt{\delta} - (1-\kappa\lambda)\sqrt{\delta}'\right]T \\ + \left[\left((1-\kappa\lambda)\,\kappa + \lambda'' - \lambda\tau^2\right)\sqrt{\delta} - \lambda'\sqrt{\delta}'\right]N \\ + \left[(\lambda'\tau + (\tau\lambda)')\sqrt{\delta} - \tau\lambda\sqrt{\delta}'\right]B \end{array}\right].$$

Also $\kappa^{*2} = \langle \kappa^* N^*, \kappa^* N^*\rangle$, so we have

$$\kappa^* = \frac{1}{\sqrt{\delta^3}}\left(\begin{array}{c} \left((\kappa'\lambda + 2\lambda'\kappa)\sqrt{\delta} + (1-\kappa\lambda)\sqrt{\delta}'\right)^2 + \left(\left(\kappa - \lambda\left(\kappa^2+\tau^2\right)+\lambda''\right)\sqrt{\delta} - \lambda'\sqrt{\delta}'\right)^2 \\ + \left((2\tau'\lambda + \tau\lambda')\sqrt{\delta} - \lambda\tau\sqrt{\delta}'\right)^2 \end{array}\right)^{\frac{1}{2}}.$$

For $N - D^*$ *partner* curve, under the condition $\dfrac{\lambda'}{\cos\theta} = \sqrt{\delta}$ and $2\left(\dfrac{\lambda'}{\cos\theta}\right)'\dfrac{\lambda'}{\cos\theta} = \delta'$ we have the proof. □

2.3. *Normal vector field of N − D* partner curve*

Theorem 2.7. *If the normal vector field of $N - D^*$ partner curve is N^*, then it can be given based on the Frenet apparatus of the first curve as*

$$N^* = \frac{1}{\nabla}\left(\begin{array}{c} \left((\kappa'\lambda + 2\lambda'\kappa)\sqrt{\delta} + (1-\kappa\lambda)\sqrt{\delta}'\right)T + \left(\left(\kappa - \lambda\left(\kappa^2+\tau^2\right)+\lambda''\right)\sqrt{\delta} - \lambda'\sqrt{\delta}'\right)N \\ + \left((2\tau'\lambda + \tau\lambda')\sqrt{\delta} - \lambda\tau\sqrt{\delta}'\right)B \end{array}\right),$$

where

$$\nabla=\begin{pmatrix} \left((1-\kappa\lambda)'\sqrt{\delta}-\lambda'\kappa-(1-\kappa\lambda)\sqrt{\delta'}\right)^2+\left(\left(\kappa-\lambda\left(\kappa^2+\tau^2\right)+\lambda''\right)\sqrt{\delta}-\lambda'\sqrt{\delta'}\right)^2 \\ +\left((\lambda'\tau+(\tau\lambda)')\sqrt{\delta}-\tau\lambda\sqrt{\delta'}\right)^2 \end{pmatrix}^{\frac{1}{2}}.$$

Proof. Since $\kappa^* N^* = \frac{dT^*}{ds}\frac{ds}{ds^*}$, we have the general form as following:

$$\kappa^* N^* = \frac{1}{\sqrt{\delta^3}}\begin{pmatrix} \left((\kappa'\lambda+2\lambda'\kappa)\sqrt{\delta}+(1-\kappa\lambda)\sqrt{\delta'}\right)T+\left(\left(\kappa-\lambda\left(\kappa^2+\tau^2\right)+\lambda''\right)\sqrt{\delta}-\lambda'\sqrt{\delta'}\right)N \\ +\left((2\tau'\lambda+\tau\lambda')\sqrt{\delta}-\lambda\tau\sqrt{\delta'}\right)B \end{pmatrix}. \tag{1}$$

Under the condition that $\frac{1}{\sqrt{\delta}}=\frac{\cos\theta}{\lambda'}$, we have

$$N^*=\frac{1}{2\nabla\sqrt{\delta}}\begin{bmatrix} ((\kappa'\lambda+2\lambda'\kappa)2\delta+(1-\kappa\lambda)\delta')T+\left(\left(\kappa-\lambda\left(\kappa^2+\tau^2\right)+\lambda''\right)2\delta-\lambda'\delta'\right)N \\ +((2\tau'\lambda+\tau\lambda')2\delta-\lambda\tau\delta')B \end{bmatrix},$$

where

$$\nabla=\begin{pmatrix} \left((\kappa'\lambda+2\lambda'\kappa)\sqrt{\delta}+(1-\kappa\lambda)\sqrt{\delta'}\right)^2+\left(\left(\kappa-\lambda\left(\kappa^2+\tau^2\right)+\lambda''\right)\sqrt{\delta}-\lambda'\sqrt{\delta'}\right)^2 \\ +\left((2\tau'\lambda+\tau\lambda')\sqrt{\delta}-\lambda\tau\sqrt{\delta'}\right)^2 \end{pmatrix}^{\frac{1}{2}},$$

which completes the proof. □

2.4. Binormal vector field of $N-D^*$ partner curve

Theorem 2.8. *If the binormal vector field of $N-D^*$ partner curve is B^*, then it can be given based on the Frenet apparatus of the first curve as*

$$B^*=\frac{1}{\nabla}\begin{pmatrix} \left(\lambda^2\tau^3+\lambda'(\lambda\tau)'+\tau(\lambda')^2+\kappa^2\lambda^2\tau-\kappa\lambda\tau-\lambda\tau\lambda''\right)T \\ +(-((\lambda\tau)'+\tau\lambda'-\lambda\tau(1-\kappa\lambda)'-\kappa\lambda(\lambda\tau)'))N \\ +\left(\kappa+\lambda''-\lambda'(1-\kappa\lambda)'+\kappa^3\lambda^2-2\kappa^2\lambda-\lambda\tau^2+\kappa(\lambda')^2+\kappa\lambda^2\tau^2-\kappa\lambda\lambda''\right)B \end{pmatrix}.$$

Proof. It is clear that $B^*=T^*\Lambda N^*$, hence

$$B^*=\frac{1}{\nabla\sqrt{\delta}}\begin{pmatrix} \left(\sqrt{\delta}\left(\lambda^2\tau^3+\lambda'(\lambda\tau)'+\tau(\lambda')^2+\kappa^2\lambda^2\tau-\kappa\lambda\tau-\lambda\tau\lambda''\right)\right)T \\ +\left(-\sqrt{\delta}((\lambda\tau)'+\tau\lambda'-\lambda\tau(1-\kappa\lambda)'-\kappa\lambda(\lambda\tau)')\right)N \\ +\left(\sqrt{\delta}\left(\kappa+\lambda''-\lambda'(1-\kappa\lambda)'+\kappa^3\lambda^2-2\kappa^2\lambda-\lambda\tau^2+\kappa(\lambda')^2+\kappa\lambda^2\tau^2-\kappa\lambda\lambda''\right)\right)B \end{pmatrix}$$

$$=\frac{1}{\nabla\sqrt{\delta}}\begin{pmatrix} \left(\sqrt{\delta}\left(\lambda^2\tau^3+\lambda'(\lambda\tau)'+\tau(\lambda')^2+\kappa^2\lambda^2\tau-\kappa\lambda\tau-\lambda\tau\lambda''\right)\right)T \\ +\left(-\sqrt{\delta}((\lambda\tau)'+\tau\lambda'-\lambda\tau(1-\kappa\lambda)'-\kappa\lambda(\lambda\tau)')\right)N \\ +\left(\sqrt{\delta}\left(\kappa+\lambda''-\lambda'(1-\kappa\lambda)'+\kappa^3\lambda^2-2\kappa^2\lambda-\lambda\tau^2+\kappa(\lambda')^2+\kappa\lambda^2\tau^2-\kappa\lambda\lambda''\right)\right)B \end{pmatrix}.$$

□

Corollary 2.9. *There is the relationship among the curvatures of* $N-D^*$ *curve* λ*,and angle* θ *based on the Frenet-Serret apparatus as in the following way*

$$\nabla\theta'\cos\theta = \kappa\tau + \tau\lambda'' - \lambda\tau^3 - \kappa^2\lambda\tau - \kappa\lambda\lambda'\tau' + \lambda\tau\kappa'\lambda'.$$

Proof. We know that $\langle N, B^*\rangle = \langle D^*, B^*\rangle = \|B^*\|\,\|D^*\|\cos\left(\frac{\pi}{2}-\theta\right) = \sin\theta$, and derivation of both sides give us

$$\begin{aligned}\langle N', B^*\rangle + \left\langle N, B^{*'}\right\rangle &= \theta'\cos\theta,\\ \langle -\kappa T + \tau B, B^*\rangle + \left\langle N, -\tau^* N^*\frac{ds^*}{ds}\right\rangle &= \theta'\cos\theta,\\ -\kappa\langle T, B^*\rangle + \tau\langle B, B^*\rangle &= \theta'\cos\theta.\end{aligned}$$

As a result we have

$$\begin{aligned}\nabla\sqrt{\delta}\theta'\cos\theta = &-\kappa\left(\sqrt{\delta}\left(\lambda^2\tau^3 + \lambda'(\lambda\tau)' + \tau(\lambda')^2 + \kappa^2\lambda^2\tau - \kappa\lambda\tau - \lambda\tau\lambda''\right)\right)\\ &+\tau\left(\sqrt{\delta}\left(\kappa + \lambda'' - \lambda'(1-\kappa\lambda)' + \kappa^3\lambda^2 - 2\kappa^2\lambda - \lambda\tau^2 + \kappa(\lambda')^2 + \kappa\lambda^2\tau^2 - \kappa\lambda\lambda''\right)\right).\end{aligned}$$

Hence the result of these products completes the proof with the equality

$$-\sqrt{\delta}\left(\lambda\tau^3 - \tau\lambda'' - \kappa\tau + \kappa^2\lambda\tau + \tau\lambda'(1-\kappa\lambda)' + \kappa\lambda'(\lambda\tau)'\right) = \nabla\sqrt{\delta}\theta'\cos\theta,$$

$$\nabla\theta'\cos\theta = -\left(\lambda\tau^3 - \tau\lambda'' - \kappa\tau + \kappa^2\lambda\tau + \tau\lambda'(1-\kappa\lambda)' + \kappa\lambda'(\lambda\tau)'\right).$$

□

2.5. *Second curvature of* $N-D^*$ *partner curve*

Theorem 2.10. *If the second curvature of* $N-D^*$ *partner curve is* τ^**, then it can be given based on the Frenet apparatus of the first curve as*

$$\tau^* = \frac{\nabla^2\lambda'\cos\theta + \left(2\kappa^2\lambda' - \tau^2\lambda' + \kappa\lambda\kappa' - 2\lambda\tau\tau'\right)\lambda'\sqrt{\delta^3} + \left(\kappa - \kappa^2\lambda + \lambda\tau^2\right)\cos\theta\sqrt{\delta^3}\sqrt{\delta}'}{\sqrt{\delta^3}\,(2\lambda'\tau + \lambda\tau' - \lambda^2\kappa\tau')\,\lambda'}.$$

Proof. Since the definition of $N-D^*$ partner curve we know that $\langle N^*, N\rangle = 0$. So derivation of both sides give us

$$\left\langle \frac{dN^*}{ds}, N\right\rangle + \langle N^*, N'\rangle = 0,$$

$$\left\langle (-\kappa^* T^* + \tau^* B^*)\frac{ds^*}{ds}, N\right\rangle + \langle N^*, N'\rangle = 0.$$

As a result we have

$$\tau^*\langle B^*, N\rangle = \kappa^*\langle T^*, N\rangle - \frac{1}{\sqrt{\delta}}\langle N^*, -\kappa T + \tau B\rangle.$$

Hence the result of these products completes the proof with the equality

$$\tau^* = \frac{\kappa^*\nabla\lambda'\cos\theta + \left(2\kappa^2\lambda' - \tau^2\lambda' + \kappa\lambda\kappa' - 2\lambda\tau\tau'\right)\lambda' + \left(\kappa - \kappa^2\lambda + \lambda\tau^2\right)\cos\theta\sqrt{\delta}'}{(2\lambda'\tau + \lambda\tau' - \lambda^2\kappa\tau')\,\lambda'}.$$

□

3. Conclusion

In this study the principal normal vector of the curve α and unit Darboux vector of second curve β have been taken linearly dependent, then we get a new partner curve wich has been called $N - D^*$ curve as a way of generate the new curves. Also Frenet-Serret apparatus of $N - D^*$ curve have been given based on the Frenet-Serret apparatus of first curve α. In a similar way, using alternative frame vectors new associated curves can be defined. Further, Frenet-Serret apparatus of these curves can be given based on the Frenet-Serret apparatus of first curve α.

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