TURKISH JOURNAL OF SCIENCE
VOLUME 9, ISSUE 2, 157–161
ISSN: 2587–0971

https://www.tjoscience.com

Some Notes on Semi-Tensor Bundle

Furkan YILDIRIM[a], Merve AYDIN[b]

[a]*Narman Vocational Training School, Ataturk University, 25530, Erzurum, Turkey*
[b]*Department of Mathematics, Faculty of Sci. Ataturk University, 25240, Erzurum, Turkey*

Abstract. Using the fiber bundle M over a manifold B, we define a semi-tensor (pull-back) bundle tB of type (p,q). The present paper is devoted to some results concerning with the horizontal lifts of some tensor fields from manifold B to its semi-tensor bundle tB of type (p,q).

1. Introduction

Let M_n be an n-dimensional differentiable manifold of class C^∞ and $\pi_1 : M_n \to B_m$ the differentiable bundle determined by a submersion π_1. Suppose that $(x^i) = (x^a, x^\alpha)$, $a, b, ... = 1, ..., n-m; \alpha, \beta, ... = n-m+1, ..., n; i, j, ... = 1, 2, ..., n$ is a system of local coordinates adapted to the bundle $\pi_1 : M_n \to B_m$, where x^α are coordinates in B_m, and x^a are fiber coordinates of the bundle $\pi_1 : M_n \to B_m$. If $(x^{i'}) = (x^{a'}, x^{\alpha'})$ is another system of local adapted coordinates in the bundle, then we have [8]

$$\begin{cases} x^{a'} = x^{a'}\left(x^b, x^\beta\right), \\ x^{\alpha'} = x^{\alpha'}\left(x^\beta\right). \end{cases} \tag{1}$$

The Jacobian of (1) has components

$$\left(A_j^{i'}\right) = \left(\frac{\partial x^{i'}}{\partial x^j}\right) = \begin{pmatrix} A_b^{a'} & A_\beta^{a'} \\ 0 & A_\beta^{\alpha'} \end{pmatrix},$$

where

$$A_\beta^{\alpha'} = \frac{\partial x^{\alpha'}}{\partial x^\beta}.$$

Let $\left(T_q^p\right)_x (B_m)(x = \pi_1(\widetilde{x}), \widetilde{x} = (x^a, x^\alpha) \in M_n)$ be the tensor space at a point $x \in B_m$ with local coordinates $(x^1, ..., x^m)$, we have the holonomous frame field

$$\partial_{x^{i_1}} \otimes \partial_{x^{i_2}} \otimes ... \otimes \partial_{x^{i_p}} \otimes dx^{j_1} \otimes dx^{j_2} \otimes ... \otimes dx^{j_q},$$

Corresponding author: FY mail address: furkan.yildirim@atauni.edu.tr ORCID: 0000-0003-0081-7857, MA ORCID:0000-0003-0825-7196

Received: 15 July 2024; Accepted: 19 September 2024; Published: 30 September 2024.
Keywords. Vector field, horizontal lift, pull-back bundle, semi-tensor bundle
2010 *Mathematics Subject Classification*. 53A45, 55R10, 57R25
Cited this article as: Yıldırım, F. & Aydın, M. (2014). Some Notes on Semi-Tensor Bundle. Turkish Journal of Science, 9(2), 157–161.

for $i \in \{1,...,m\}^p$, $j \in \{1,...,m\}^q$, over $U \subset B_m$ of this tensor bundle, and for any (p,q)-tensor field t we have [[4], p.163]:

$$t|\,U = t^{i_1...i_p}_{j_1...j_q}\partial_{x^{i_1}} \otimes \partial_{x^{i_2}} \otimes ... \otimes \partial_{x^{i_p}} \otimes dx^{j_1} \otimes dx^{j_2} \otimes ... \otimes dx^{j_q},$$

then by definition the set of all points $(x^I) = (x^a, x^\alpha, x^{\overline{\alpha}})$, $x^{\overline{\alpha}} = t^{i_1...i_p}_{j_1...j_q}$, $\overline{\alpha} = \alpha + m^{p+q}$, $I, J, ... = 1, ..., n + m^{p+q}$ is a semi-tensor bundle $t^p_q(B_m)$ over the manifold M_n [8], [15], [18] . The semi-tensor bundle $t^p_q(B_m)$ has the natural bundle structure over B_m, its bundle projection $\pi : t^p_q(B_m) \to B_m$ being defined by $\pi : (x^a, x^\alpha, x^{\overline{\alpha}}) \to (x^\alpha)$. If we introduce a mapping $\pi_2 : t^p_q(B_m) \to M_n$ by $\pi_2 : (x^a, x^\alpha, x^{\overline{\alpha}}) \to (x^a, x^\alpha)$, then $t^p_q(B_m)$ has a bundle structure over M_n. It is easily verified that $\pi = \pi_1 \circ \pi_2$ [6],[8],[15], [18].

On the other hand, let $\varepsilon = \pi : E \to B$ denote a fiber bundle with fiber F. Given a manifold B' and a map $f : B' \to B$, one can construct in a natural way a bundle over B' with the same fiber: Consider the subset

$$f^*E = \{(b', e) \in B' \times E |\, f(b') = \pi(e)\}$$

together with the subspace topology from $B' \times E$, and denote by $\pi_1 : f^*E \to B'$, $\pi_2 : f^*E \to E$ the projections. $f^*\varepsilon = \pi_1 : f^*E \to B'$ is a fiber bundle with fiber F, called the pull-back bundle of ε via f [[3], [5], [8], [9], [11], [14], [15], [18]].

From the above definition it follows that the semi-tensor bundle $(t^p_q(B_m), \pi_2)$ is a pull-back bundle of the tensor bundle over B_m by π_1 (see, for example [8], [13], [15], [18]).

In other words, the semi-tensor bundle (induced or pull-back bundle) of the tensor bundle $\left(T^p_q(B_m), \widetilde{\pi}, B_m\right)$ is the bundle $\left(t^p_q(B_m), \pi_2, M_n\right)$ over M_n with a total space $t^p_q(B_m) = \left\{((x^a, x^\alpha), x^{\overline{\alpha}}) \in M_n \times \left(T^p_q\right)_x (B_m) : \pi_1(x^a, x^\alpha) = \widetilde{\pi}\left(x^\alpha, x^{\overline{\alpha}}\right) = (x^\alpha\right.$ $M_n \times \left(T^p_q\right)_x (B_m)$.To a transformation (1) of local coordinates of M_n, there corresponds on $t^p_q(B_m)$ the coordinate transformation

$$\begin{cases} x^{a'} = x^{a'}\left(x^b, x^\beta\right), \\ x^{\alpha'} = x^{\alpha'}\left(x^\beta\right), \\ x^{\overline{\alpha}'} = t^{\beta'_1...\beta'_p}_{\alpha'_1...\alpha'_q} = A^{\beta'_1...\beta'_p}_{\alpha_1...\alpha_p} A^{\beta_1...\beta_q}_{\alpha'_1...\alpha'_q} t^{\alpha_1...\alpha_p}_{\beta_1...\beta_q} = A^{(\beta')}_{(\alpha)} A^{(\beta)}_{(\alpha')} x^{\overline{\beta}}. \end{cases} \tag{2}$$

The Jacobian of (2) is given by [8], [15], [18]:

$$\bar{A} = \left(A^{I'}_J\right) = \begin{pmatrix} A^{a'}_b & A^{a'}_\beta & 0 \\ 0 & A^{\alpha'}_\beta & 0 \\ 0 & t^{(\alpha)}_{(\sigma)}\partial_\beta A^{(\beta')}_{(\alpha)} A^{(\sigma)}_{(\alpha')} & A^{(\beta')}_{(\alpha)} A^{(\beta)}_{(\alpha')} \end{pmatrix}, \tag{3}$$

where $I = (a, \alpha, \overline{\alpha})$, $J = (b, \beta, \overline{\beta})$, $I, J... = 1, ..., n + m^{p+q}$, $t^{(\alpha)}_{(\sigma)} = t^{\alpha_1...\alpha_p}_{\sigma_1...\sigma_q}$, $A^{\alpha'}_\beta = \frac{\partial x^{\alpha'}}{\partial x^\beta}$.

It is easily verified that the condition $Det\bar{A} \neq 0$ is equivalent to the condition:

$$Det(A^{a'}_b) \neq 0, Det(A^{\alpha'}_\beta) \neq 0, Det(A^{(\beta')}_{(\alpha)} A^{(\beta)}_{(\alpha')}) \neq 0.$$

Also, $\dim t^p_q(B_m) = n + m^{p+q}$. In the special case $n=m$, $t^p_q(B_m)$ is a tensor bundle $T^p_q(B_m)$ [[6], p.118]. In the special case, the semi-tensor bundles $t^1_0(B_m)$ $(p = 1,\ q = 0)$ and $t^0_1(B_m)$ $(p = 0,\ q = 1)$ are semi-tangent and semi-cotangent bundles, respectively. We note that semi-tangent and semi-cotangent bundle were examined in [[1], [7], [10]] and [[12], [14], [16], [17]], respectively. Also, Fattaev studied the special class of semi-tensor bundle [2]. We denote by $\Im^p_q(t^p_q(B_m))$ and $\Im^p_q(B_m)$ the modules over $F\left(t^p_q(B_m)\right)$ and $F(B_m)$ of all tensor fields of type (p,q) on $t^p_q(B_m)$ and B_m respectively, where $F\left(t^p_q(B_m)\right)$ and $F(B_m)$ denote the rings of real-valued C^∞ −functions on $t^p_q(B_m)$ and B_m, respectively.

2. Some lifts of tensor fields and $\gamma-$ Operator

Let $\widetilde{X} \in \Im_0^1(M_n)$ be a projectable vector field [10] with projection $X = X^\alpha(X^\alpha)\partial_\alpha$ i.e. $\widetilde{X} = \widetilde{X}^a(x^a, x^\alpha)\partial_a + X^\alpha(x^\alpha)\partial_\alpha$. On putting

$$ {}^{cc}\widetilde{X} = \begin{pmatrix} {}^{cc}\widetilde{X}^b \\ {}^{cc}X^\beta \\ {}^{cc}X^{\bar\beta} \end{pmatrix} = \begin{pmatrix} \widetilde{X}^b \\ X^\beta \\ \sum_{\lambda=1}^{p} t_{\beta_1\ldots\beta_q}^{\alpha_1\ldots\epsilon\ldots\alpha_p}\partial_\varepsilon X^{\beta_\lambda} - \sum_{\mu=1}^{q} t_{\beta_1\ldots\epsilon\ldots\ldots\beta_q}^{\alpha_1\ldots\alpha_p}\partial_{\beta_\mu}X^\varepsilon \end{pmatrix} \tag{4}$$

we easily see that ${}^{cc}\widetilde{X}\,' = \bar{A}\left({}^{cc}\widetilde{X}\right)$. The vector field ${}^{cc}\widetilde{X}$ is called the complete lift of $\widetilde{X}$ to the semi-tensor bundle $t_q^p(B_m)$ [15].

Now, consider $A \in \Im_q^p(B_m)$ and $\varphi \in \Im_1^1(B_m)$, then ${}^{w}A \in \Im_0^1\left(t_q^p(B_m)\right)$ (vertical lift), $\gamma\varphi \in \Im_0^1\left(t_q^p(B_m)\right)$ and $\gamma\varphi \in \Im_0^1\left(t_q^p(B_m)\right)$ have respectively, components on the semi-tensor bundle $t_q^p(B_m)$ [15]

$$ {}^{w}A = ({}^{w}A)^I = \begin{pmatrix} {}^{w}A^a \\ {}^{w}A^\alpha \\ {}^{w}A^{\bar\alpha} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ A_{\beta_1,\beta_q}^{\alpha_p,\ldots} \end{pmatrix}, \gamma\varphi = (\gamma\varphi)^I = \begin{pmatrix} 0 \\ 0 \\ \sum_{\lambda=1}^{p} t_{\beta_1\ldots\beta_q}^{\alpha_1\ldots\ldots\ldots\alpha_p}\varphi_\varepsilon^{\alpha_\lambda} \end{pmatrix}, \tag{5}$$

On the other hand, ${}^{vv}f$ the vertical lift of function $f \in \Im_0^0(B_m)$ on $t_q^p(B_m)$ is defined by [15]:

$$ {}^{w}f = {}^{v} f \circ \pi_2 = f \circ \pi_1 \circ \pi_2 = f \circ \pi. \tag{6}$$

Let $S \in \mathcal{J}_2^1(B_m)$ now. If we take account of (3), we see that $\gamma S' = \bar{A}(\gamma S) \cdot \gamma S$ is given by

$$ \gamma S = \left((\gamma S)_J^I\right) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \sum_{\lambda=1}^{p} S_{\beta_\varepsilon}^{\beta_\lambda}\xi_{\alpha_1\ldots\alpha_q}^{\beta_1\ldots\beta_p} & 0 \end{pmatrix}, \tag{7}$$

with respect to the coordinates $(x^a, x^\alpha, x^{\bar\alpha})$ on $t_q^p(B_m)$, where $S_{\beta_\varepsilon}^{\beta_\lambda}$ are local components of S.

Let $S \in \Im_2^1(B_m)$ now. If we take account of (3), we see that $\gamma S' = \bar{A}(\gamma S) \cdot \gamma S$ is given by

$$ \gamma S = \left((\gamma S)_J^I\right) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \sum_{\mu=1}^{q} S_{\beta_\lambda}^{\beta_\varepsilon}\xi_{\beta_1\ldots\varepsilon\ldots\beta_q}^{\alpha_1\ldots\alpha_p} & 0 \end{pmatrix} \tag{8}$$

with respect to the coordinates $(x^a, x^\alpha, x^{\bar\alpha})$ on $t_q^p(B_m)$, where $S_{\beta_\lambda}^{\beta_\varepsilon}$ are local components of S.

3. Horizontal lifts of vector fields and $\gamma-$Operator

Let $\widetilde{X} \in \Im_0^1(M_n)$ be a projectable vector field [10] with projection $X = X^\alpha(x^\alpha)\partial_\alpha$ i.e. $\widetilde{X} = \widetilde{X}^a(x^a, x^\alpha)\partial_a + X^\alpha(x^\alpha)\partial_\alpha$. If we take account of (3), we can prove that ${}^{HH}\widetilde{X}' = \bar{A}\left({}^{HH}\widetilde{X}\right)$, where ${}^{HH}\widetilde{X}$ is a vector field defined by

$$ {}^{HH}\widetilde{X} = \begin{pmatrix} \widetilde{X}^b \\ X^\beta \\ X^l\left(\sum_{\mu=1}^{q} \Gamma_{l\beta_\mu}^{\varepsilon} t_{\beta_1\ldots\varepsilon\ldots\beta_q}^{\alpha_1\ldots\alpha_p} - \sum_{\lambda=1}^{p} \Gamma_{l\varepsilon}^{\beta_\lambda} t_{\beta_1\ldots\beta_q}^{\alpha_1\ldots\varepsilon\ldots\alpha_p}\right) \end{pmatrix}, \tag{9}$$

with respect to the coordinates $(x^b, x^\beta, x^{\bar\beta})$ on $t_q^p(B_m)$. We call ${}^{HH}\widetilde{X}$ the horizontal lift of the vector field of the vector field $\widetilde{X}$ to $t_q^p(B_m)$ [18].

Theorem 3.1. *For any vector fields X on M_n and $S,T \in \Im_2^1(B_m)$, $\varphi \in \Im_1^1(B_m)$, $A \in \Im_q^p(B_m)$, we have*
(i) $(\gamma S)^{cc}X = \gamma(S_X)$,
(ii) $(\gamma S)({}^{v}A) = 0$
(iii) $(\gamma S)(\gamma\varphi) = 0$
(iv) $(\gamma S)(\gamma T) = 0$.

Proof. (*i*) Using (4) and (7), we have

$$(\gamma S)^{cc}X = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \sum_{\lambda=1}^{p} S_{\beta_\varepsilon}^{\beta_\lambda}\xi_{\alpha_1\ldots\alpha_q}^{\beta_1\ldots\ldots\beta_p} & 0 \end{pmatrix}\begin{pmatrix} X^a \\ \sum^{\alpha} \\ \sum_{\lambda=1}^{p} t_{\alpha_1\ldots\alpha_q}^{\phi_1\ldots\ldots\phi_p}\partial_\varepsilon X^{\alpha_\lambda} - \sum_{\mu=1}^{q} t_{\alpha_1\ldots\ldots\ldots\alpha_q}^{\phi_1\ldots\phi_p}\partial_{\phi_\mu}X^{\varepsilon} \end{pmatrix}$$
$$= \begin{pmatrix} 0 \\ 0 \\ \sum_{\lambda=1}^{p} S_{\beta_\varepsilon}^{\beta_\lambda}\xi_{\alpha_1\ldots\alpha_q}^{\beta_1\ldots\ldots\beta_p}X^{\alpha} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \sum_{\lambda=1}^{p} \xi_{\alpha_1\ldots\alpha_q}^{\beta_1\ldots\varepsilon\ldots\beta_p}(S_X)_{\varepsilon}^{\beta_\lambda} \end{pmatrix} = \gamma(S_X).$$

(*ii*) Using (5) and (7), we have

$$(\gamma S)({}^{wv}A) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \sum_{\lambda=1}^{p} S_{\beta_\varepsilon}^{\beta_\lambda}\xi_{\alpha_1\ldots\alpha_q}^{\beta_1\ldots\varepsilon\ldots\beta_p} & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ A_{\beta_1\ldots\beta_q}^{\phi_1\ldots\phi_p} \end{pmatrix} = 0.$$

(*iii*) Using (5) and (7), we have

$$(\gamma S)(\gamma\varphi) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \sum_{\lambda=1}^{p} S_{\beta_\varepsilon}^{\beta_\lambda}\xi_{\alpha_1\ldots\alpha_q}^{\beta_1\ldots\ldots\beta_p} & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ \sum_{\lambda=1}^{p} t_{\beta_1\ldots\beta_q}^{\phi_1\ldots\varepsilon\ldots\phi_p}\varphi_{\varepsilon}^{\phi_\lambda} \end{pmatrix} = 0.$$

(*iv*) Using (7), we have

$$(\gamma S)(\gamma T) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \sum_{\lambda=1}^{p} S_{\beta_\varepsilon}^{\beta_\lambda}\xi_{\alpha_1\ldots\alpha_q}^{\beta_1\ldots\beta_p} & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \sum_{\lambda=1}^{p} T_{\phi_\varepsilon}^{\alpha_\lambda}\xi_{\phi_1\ldots\phi_q}^{\alpha_1\ldots\ldots\alpha_p} & 0 \end{pmatrix} = 0.$$

□

Theorem 3.2. *For any vector fields X on M_n and $S,T \in \Im_2^1(B_m)$, $\varphi \in \Im_1^1(B_m)$, $A \in \Im_q^p(B_m)$, we have*
(i) $(\gamma S) = \gamma(S_X)$,
(ii) $(\gamma S)({}^{vv}A) = 0$,
(iii) $(\gamma S)(\gamma\varphi) = 0$,
(iv) $(\gamma S)(\gamma T) = 0$.

Proof. Using (4), (5) and (8), similarly, we obtain Theorem 3.2. □

Theorem 3.3. *If $X \in \Im_0^1(M_n)$, $S \in \Im_2^1(B_m)$, then*

$$(\gamma S)\left({}^{HH}X\right) = \gamma(S_X).$$

Proof. By (7) and (9), we obtain

$$(\gamma S)\left({}^{HH}X\right) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \sum_{\lambda=1}^{p} S_{\beta_\varepsilon}^{\beta_\lambda}\xi_{\alpha_1\ldots\alpha_q}^{\beta_1\ldots\ldots\ldots\beta_p} & 0 \end{pmatrix}\begin{pmatrix} X^a \\ X^{\alpha} \\ X^l\left(\sum_{\mu=1}^{q}\Gamma_{l\alpha_\mu}^{\varepsilon}t_{\alpha_1\ldots\ldots\ldots\alpha_q}^{\phi_1\ldots\phi_p} - \sum_{\lambda=1}^{p}\Gamma_{l\varepsilon}^{\alpha_2}t_{\alpha_1\ldots\alpha_q}^{\phi_1\ldots\ldots\phi_p}\right) \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 0 \\ \sum_{\lambda=1}^{p} S_{\beta_\varepsilon}^{\beta_\lambda} \xi_{\alpha_1 \ldots \alpha_q}^{\beta_1 \ldots\ldots \beta_p} X^\alpha \end{pmatrix} = \begin{pmatrix} 0 \\ \sum_{\lambda=1}^{p} \xi_{\alpha_1 \ldots \alpha_q}^{\beta_1 \ldots\ldots \beta_p} (S_X)_\varepsilon^{\beta_\lambda} \end{pmatrix} = \gamma (S_X).$$

□

Theorem 3.4. *If* $X \in \Im_0^1 (M_n)$, $S \in \Im_2^1 (B_m)$, *then*

$$(\widetilde{\gamma} S) \left({}^{HH}X \right) = \widetilde{\gamma} (S_X).$$

Proof. Using (8) and (9), similarly, we have Theorem 3.4. □

References

[1] T.V. Duc, Structure presque-transverse. J. Diff. Geom., 14 (1979), no. 2, 215-219.
[2] H. Fattaev, The Lifts of Vector Fields to the Semitensor Bundle of the Type (2,0), Journal of Qafqaz University, 25 (2009), no. 1, 136-140.
[3] D. Husemoller, Fibre Bundles. Springer, New York, 1994.
[4] V. Ivancevic and T. Ivancevic, Applied Differential Geometry, A Modern Introduction, World Scientific, Singapore, 2007.
[5] H.B. Lawson and M.L. Michelsohn, Spin Geometry. Princeton University Press., Princeton, 1989.
[6] A. Salimov, Tensor Operators and their Applications. Nova Science Publ., New York, 2013.
[7] A. A. Salimov and E. Kadıoğlu, Lifts of derivations to the semitangent bundle, Turk J. Math. 24 (2000), no. 3, 259-266.
[8] M. Polat and N. Cengiz, Some properties of semi-tensor bundle, New Trends in Mathematical Sciences, 6 (2018), no. 3, 147-153.
[9] N. Steenrod, The Topology of Fibre Bundles. Princeton University Press., Princeton,1951.
[10] V. V. Vishnevskii, Integrable affinor structures and their plural interpretations. Geometry, 7.J. Math. Sci. (New York) 108 (2002), no. 2, 151-187.
[11] G. Walschap, Metric Structures in Differential Geometry, Graduate Texts in Mathematics, Springer-Verlag, New York, 2004.
[12] F. Yıldırım, On a special class of semi-cotangent bundle, Proc. Inst. Math. Mech. Natl. Acad. Sci. Azerb., 41 (2015), no. 1, 25-38.
[13] F. Yıldırım, A pull-Back bundle of tensor bundles defined by projection of the tangent bundle, Ordu University Journal of Science and Technology, 7 (2017), no. 2, 353-366.
[14] F. Yıldırım, Complete lift of a tensor field of type (1,2) to semi-cotangent bundle, New Trends in Mathematical Sciences, 5 (2017), no. 4, 261-270.
[15] F. Yıldırım, Note on the cross-section in the semi-tensor bundle, New Trends in Mathematical Sciences, 5 (2017), no. 2, 212-221.
[16] F. Yildirim, M.B. Asl, F. Jabrailzade, Vector and affinor fields on cross-sections in the semi-cotangent bundle, Proc. Inst. Math. Mech. Natl. Acad. Sci. Azerb., 43 (2017), no. 2, 305-315.
[17] F. Yıldırım and A. Salimov, Semi-cotangent bundle and problems of lifts. Turk J Math, 38 (2014), no.2, 325-339.
[18] F. Yıldırım, Horizontal Lift in the Semi-Tensor Bundle. Konuralp Journal of Mathematics, 6 (2018), no.2, 338-344.