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İkinci Mertebeden Kompleks Aileler için Ortak Diyagonal Çözümler

Common Diagonal Solutions for Second Order Complex Family

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Öz: Bu çalışmada ikinci mertebeden kompleks aralık matris aileleri için Lyapunov eşitsizliğinin ortak diyagonal çözümlerinin varlığı ve değerlendirilmesi problemi ele alınmıştır. Ele alınan probleme dair gerek ve yeter koşullar verilmiştir. Ayrıca bilinen yeterli koşulların ortak çözümleri vermediği durumlar için örnekler verilmiştir.
Anahtar Kelimeler — Kompleks aralık matris; Lyapunov eşitsizliği; Diyagonal kararlılık; Ortak Diyagonal çözüm.

Abstract: In this paper for second order complex interval matrix family we consider the problem of existence and evaluation of common diagonal solutions for the Lyapunov matrix inequality. Necessary and sufficient conditions are given. Numbers of examples are given, where known sufficient conditions do not give common solutions.
Keywords — Complex Interval Matrices; Lyapunov Inequality; Diagonal Stability; Common Diagonal Solution

1. Introduction

Let a 2×2 complex interval matrix family

$$\mathsf{A}=\left\{\begin{bmatrix} a_1+j\widetilde{a}_1 & a_2+j\widetilde{a}_2 \\ a_3+j\widetilde{a}_3 & a_4+j\widetilde{a}_4 \end{bmatrix} : a_i^- \le a_i \le a_i^+, \widetilde{a}_i^- \le \widetilde{a}_i \le \widetilde{a}_i^+ \; i=1,\dots,4\right\} \tag{1}$$

be given. Define the following 8-dimensional box

$$Q=[a_1^-,a_1^+]\times\cdots\times[a_4^-,a_4^+]\times[\widetilde{a}_1^-,\widetilde{a}_1^+]\times\cdots\times[\widetilde{a}_4^-,\widetilde{a}_4^+] \tag{2}$$

For $\lambda_1>0$, $\lambda_2>0$ define positive definite diagonal matrix $D=\mathrm{diag}(\lambda_1,\lambda_2)$.

Definition 1. If there exists positive diagonal D such that for all $A\in\mathsf{A}$

$$A^*D+DA<0, \tag{3}$$

where A^* is the conjugate transpose of A, that is $A^*=(\overline{A})^T$, the matrix D is called common diagonal solution to the Lyapunov matrix inequality (3). In the above the symbol "$<$" stands for the negative definiteness.

Diagonal stability problems of dynamical systems have many applications in economics, large scale systems, neural networks, see [1] and the references therein. The existence problems of the diagonal type solutions for different systems are considered in [1-16]. Among them it should be noted the works [1, 2, 7, 9, 13-16], more related to our present work. The monograph [1] presents a collection of results, observations on the results on diagonal stability and diagonal type Lyapunov functions. [2] gives sufficient conditions for the common diagonal stability of real interval matrices. [7] contains simple criterions of diagonal stability for second and third order matrices. In [9] an algorithm is presented for the checking of diagonal stability of a single real matrix. The algorithm is iterative and requires solving a linear programming problem at each step. In [13] a necessary and sufficient condition for the existence of a common diagonal solution of a pair of positive matrices is given.

In [14] the same problem has been solved for second and third order real interval matrix families. [14] contains a sufficient condition in the general case as well. For general complex interval families the same problem is studied in [15]. For third order real matrix polytopes the common diagonal solution problem is considered in [16]. For the general case the problem is solved by the cutting-plane algorithm. In the above mentioned works criterions for the existence and evaluation of common diagonal solutions for second order complex interval families did not considered. Our present work aims to fill this gap.

In [15], two sufficient conditions for common diagonal stability of complex interval systems are given. The first sufficient condition is stated as follows. Given $n \times n$ complex interval family $\mathcal{B} = [a_{ij}] + j[\tilde{a}_{ij}]$, where $a_{ij}^- \le a_{ij} \le a_{ij}^+$, $\tilde{a}_{ij}^- \le \tilde{a}_{ij} \le \tilde{a}_{ij}^+$ $(i, j = 1, 2, \dots, n)$ define the real matrix $U = [u_{ij}]$, where

$$u_{ij} = \begin{cases} a_{ii}^+, & \text{if } i = j \\ \max\{|a_{ij}^- + j\tilde{a}_{ij}^-|, |a_{ij}^- + j\tilde{a}_{ij}^+|, |a_{ij}^+ + j\tilde{a}_{ij}^-|, |a_{ij}^+ + j\tilde{a}_{ij}^+|\}, & \text{if } i \ne j. \end{cases} \tag{4}$$

Theorem 1 ([15]). *The family $\mathcal{B}$ has a common diagonal solution if $\rho^H(U) < 0$, where $\rho^H(U) = \max\{\mathrm{Re}(\lambda_i(U)) : i = 1, 2, \dots, n\}$.*

The formulation of the second theorem is cumbersome, therefore we do not give here its formulation. As noted in [15] the second theorem cannot be applied if

$$\rho^H\left(\begin{bmatrix} A_0 & \tilde{A}_0 \\ -\tilde{A}_0 & A_0 \end{bmatrix}\right) + \rho^H\left(\begin{bmatrix} R_A & R_{\tilde{A}} \\ R_{\tilde{A}} & R_A \end{bmatrix}\right) \ge 0, \tag{5}$$

where $A_0 = \left(\frac{a_{ij}^- + a_{ij}^+}{2}\right)$, $\tilde{A}_0 = \left(\frac{\tilde{a}_{ij}^- + \tilde{a}_{ij}^+}{2}\right)$, $R_A = \left(\frac{a_{ij}^+ - a_{ij}^-}{2}\right)$, $R_{\tilde{A}} = \left(\frac{\tilde{a}_{ij}^+ - \tilde{a}_{ij}^-}{2}\right)$.

In this paper for the family (1), we consider the problem of existence and evaluation of common diagonal solutions of Lyapunov inequality (3). We give necessary and sufficient conditions for the existence of common diagonal solutions. Number of examples are given, where common diagonal solutions are evaluated, whereas known results do not give such solutions.

The paper is organized as follows. In Section 2 we give a necessary and sufficient condition for robust diagonal stability for the second order family (1). In this stability each member is diagonally stable and each member has own diagonal solution for the Lyapunov matrix inequality (3). In Section 3 we give necessary and sufficient conditions for the existence of common diagonal solutions to the inequality (3). The cases $0 \notin [a_2^-, a_2^+] \cap [\tilde{a}_2^-, \tilde{a}_2^+]$ and $0 \in [a_2^-, a_2^+] \cap [\tilde{a}_2^-, \tilde{a}_2^+]$ are considered seperately. Number of examples are given. In Example 1 the familiy is robust diagonally stable, but there is no a common diagonal solution. In Examples 2 and 3 there are common diagonal solutions, which are obtained from Theorems 3 and 4, whereas known sufficient conditions from [15] do not give common solutions.

2. Robust diagonal stability

A necessary condition for the existence of a common diagonal solution is the robust diagonal stability. In this section we give a necessary and sufficient condition for the robust diagonal stability of the family $\mathcal{A}$.

Recall that, the family $\mathcal{A}$ is said to be robust diagonally stable if for every $A \in \mathcal{A}$ (equivalently for all $a = (a_1, \dots, a_4, \tilde{a}_1, \dots, \tilde{a}_4) \in Q$) there exists positive diagonal D such that

$$A^* D + DA < 0.$$

Proposition 2. *The family $\mathcal{A}$ (1) is robust diagonally stable if and only if*

$$a_1^+ < 0, \qquad a_4^+ < 0, \qquad \min_{a \in Q}(F^2 - 4EG) > 0, \qquad \min_{a \in Q} F > 0, \tag{6}$$

where

$$F = 4a_1a_4 - 2a_2a_3 + 2\tilde{a}_2\tilde{a}_3, \;\; E = a_2^2 + \tilde{a}_2^2, \;\; G = a_3^2 + \tilde{a}_3^2. \tag{7}$$

Proof. Without loss of generality all 2×2 positive diagonal matrices D may be normalized to have the form $D = \mathrm{diag}(t, 1)$, where $t > 0$. For $A \in \mathcal{A}$,

$$A = \begin{bmatrix} a_1 + j\tilde{a}_1 & a_2 + j\tilde{a}_2 \\ a_3 + j\tilde{a}_3 & a_4 + j\tilde{a}_4 \end{bmatrix}, \quad D = \begin{bmatrix} t & 0 \\ 0 & 1 \end{bmatrix}$$

the matrix inequality $A^*D + DA < 0$ becomes

$$A^*D + DA = \begin{bmatrix} 2a_1 t & (a_2 t + a_3) + j(\tilde{a}_2 t - \tilde{a}_3) \\ (a_2 t + a_3) - j(\tilde{a}_2 t - \tilde{a}_3) & 2a_4 \end{bmatrix} < 0$$

or

$$2a_1 t < 0, \qquad 2a_4 < 0 \qquad \text{and} \qquad \det(A^*D + DA) = -Et^2 + Ft - G > 0, \tag{8}$$

where F, E and G are defined by (7).

We are looking for conditions under which for each $a \in Q$ there exists $t > 0$ such that all three inequalities in (8) are satisfied. The first and the second are satisfied if and only if

$$a_1^+ < 0, \qquad a_4^+ < 0.$$

It is possible two cases.

Case 1. $0 \notin [a_2^-, a_2^+]$ or $0 \notin [\tilde{a}_2^-, \tilde{a}_2^+]$.

In this case $E > 0$ for all $a \in Q$. The existence of $t > 0$ such that $Et^2 - Ft + G < 0$ is equivalent to the following inequalities

$$\Delta = F^2 - 4EG > 0, \qquad \frac{F + \sqrt{\Delta}}{2E} > 0 \tag{9}$$

for all $a \in Q$. The second inequality is equivalent to

$$\min_{a \in Q} F > 0. \tag{10}$$

Indeed, if (10) is satisfied the second inequality in (9) is true. Conversely, if $F + \sqrt{\Delta} > 0$ then from $F^2 \geq F^2 - 4EG$ we have $|F| \geq \sqrt{\Delta} \Leftrightarrow F \leq -\sqrt{\Delta}$ or $F \geq \sqrt{\Delta}$. The inequality $F \leq -\sqrt{\Delta}$ is impossible due to $F + \sqrt{\Delta} > 0$. The second $F \geq \sqrt{\Delta}$ gives $F > 0$ for all $a \in Q$, that is (10) is satisfied. Consequently (6) is true.

Case 2. $0 \in [a_2^-, a_2^+]$ and $0 \in [\tilde{a}_2^-, \tilde{a}_2^+]$.

In this case the inequality $E > 0$ is not true. Define

$$Q_1 = \{a \in Q : E(a) > 0\}, \qquad Q_2 = \{a \in Q : E(a) = 0\}. \tag{11}$$

Obviously, $Q_2 = \{a \in Q : a_2 = \tilde{a}_2 = 0\}$.

If $a \in Q_1$, the statement that for all $a \in Q_1$ there exists $t > 0$ such that $Et^2 - Ft + G < 0$ is equivalent to (see Case 1)

$$\min_{a \in Q_1} (F^2 - 4EG) > 0, \qquad \min_{a \in Q_1} F > 0.$$

Additionally, $\min\limits_{a \in Q_2} (F^2 - 4EG) = \min\limits_{a \in Q_2} F^2 = (4a_1^+ a_4^+)^2 > 0$, $\min\limits_{a \in Q_2} F = 4a_1^+ a_4^+ > 0$.

If $a \in Q_2$, the statement that for all $a \in Q_2$ there exists $t > 0$ such that $Et^2 - Ft + G = -Ft + G < 0$ is satisfied automatically since $F = 4a_1 a_4 > 0$, $G \geq 0$.

In summary the conditions (6) are necessary and sufficient for robust diagonal stability.

□

3. Common diagonal solutions

In this section we give necessary and sufficient conditions for the existence of common diagonal solutions.

Theorem 3. *Let the interval family $\mathcal{A}$ (1) be given and the inequalities (6) are satisfied. Assume that $0 \notin [a_2^-, a_2^+]$ or $0 \notin [\tilde{a}_2^-, \tilde{a}_2^+]$. There exists a common diagonal solution to the Lyapunov inequalities if and only if the following inequality is satisfied*

$$\alpha := \max_{a\in Q} x_1(a) < \beta := \min_{a\in Q} x_2(a) \tag{12}$$

where

$$x_1(a) = \frac{F - \sqrt{F^2 - 4EG}}{2E}, \qquad x_2(a) = \frac{F + \sqrt{F^2 - 4EG}}{2E}. \tag{13}$$

If (12) is satisfied, for every $t \in (\alpha, \beta)$ the matrix $D = \mathrm{diag}(t, 1)$ is a common diagonal solution.

Proof. Note that under the hypothesis of the theorem $E > 0$ for all $a \in Q$.

$\Rightarrow$: Assume that there exists a common $D = \mathrm{diag}(t_*, 1)$ $(t_* > 0)$, it means that there exists $t_* > 0$ such that for any $a \in Q$

$$A^*D + DA = \begin{bmatrix} 2a_1 t_* & (a_2 t_* + a_3) + j(\tilde{a}_2 t_* - \tilde{a}_3) \\ (a_2 t_* + a_3) - j(\tilde{a}_2 t_* - \tilde{a}_3) & 2a_4 \end{bmatrix} < 0.$$

Then

$$a_1^+ < 0, \qquad a_4^+ < 0, \qquad Et_*^2 - Ft_* + G < 0$$

and for all $a \in Q$

$$x_1(a) < t_* < x_2(a).$$

where $x_1(a)$ and $x_2(a)$ defined by (13). Consequently, $\max_{a\in Q} x_1(a) < t_* < \min_{a\in Q} x_2(a)$ and (12) is satisfied.

$\Leftarrow$: Assume that (12) is satisfied and $t_* \in (\alpha, \beta)$. Then for all $a \in Q$ we have $x_1(a) < t_* < x_2(a)$ or $Et_*^2 - Ft_* + G < 0$. Consequently $D = \mathrm{diag}(t_*, 1)$ is a common solution.

□

Example 1. *Consider the following family*

$$\mathcal{A} = \begin{bmatrix} [-2,-1] + j\,[1,2] & [3,4] + j\,[1,2] \\ [-5,-1] + j\,[0.5,1] & [-2,-1.5] + j\,[1,2] \end{bmatrix}. \tag{14}$$

This family is robust diagonally stable. Indeed,

$$a_1^+ = -1 < 0, \;\; a_4^+ = -1.5 < 0,$$
$$\min_a (F^2 - 4EG) = 119 > 0, \qquad \min_a F = 13 > 0.$$

By Proposition 2, the family (14) is robust diagonally stable. On the other hand there is no a common diagonal solution, since

$$\alpha := \max_a x_1(a) = 1.0213 > \beta := \min_a x_2(a) = 0.7122$$

and by Theorem 3 there is no a common solution.

Example 2. *Consider the family*

$$\begin{bmatrix} [-10.875, 0.865] + j\,[0.8, 0.9] & [0.1, 0.2] + 0.1\,j \\ [0.5, 0.501] + j\,[0.3, 0.4] & [-0.265, -0.165] + j\,[0.1, 0.3] \end{bmatrix}$$

This family is robust diagonally stable by Proposition 2, since $a_1^+ = -0.865 < 0$, $a_4^+ = -0.165 < 0$, $\min_a(F^2 - 4EG) = 0.31636 > 0$, $\min_a F = 0.5909 > 0$. *The numbers α and β from (12) are $\alpha = 0.71249$,*

$\beta = 13.196629$. *By Theorem 3, for every* $t \in (0.71249, 13.196629)$ *the matrix* $D = \operatorname{diag}(t, 1)$ *is a common solution. Note that for this example the sufficient conditions from [15] are not satisfied.*

Indeed, (see (4))

$$U = \begin{bmatrix} -0.865 & 0.223606 \\ 0.641093 & -0.165 \end{bmatrix}, \quad \rho^H(U) = \max_i \operatorname{Re}\lambda_i(U) = 0.000609 > 0$$

and Theorem 1 fails. The second sufficient condition does not work as well, since (see (5))

$$\rho^H\left(\begin{bmatrix} A_0 & \tilde{A}_0 \\ -\tilde{A}_0 & A_0 \end{bmatrix}\right) + \rho^H\left(\begin{bmatrix} R_A & R_{\tilde{A}} \\ R_{\tilde{A}} & R_A \end{bmatrix}\right) = 4.845511 > 0.$$

Theorem 4. *Let the family* $\mathcal{A}$ *(1) be given, the inequalities (6) are satisfied and* $0 \in [a_2^-, a_2^+]$ *and* $0 \in [\tilde{a}_2^-, \tilde{a}_2^+]$. *Define*

$$\tilde{x}_1(a) = \frac{2G}{F + \sqrt{\Delta}}, \qquad \tilde{x}_2(a) = \begin{cases} \dfrac{F + \sqrt{\Delta}}{2E} & , \quad \text{if } E > 0 \\ \infty & , \quad \text{if } E = 0 \end{cases}. \tag{15}$$

Then there exists a common diagonal solution if and only if

$$\tilde{\alpha} := \max_{a \in Q} \tilde{x}_1(a) < \tilde{\beta} := \inf_{a \in Q} \tilde{x}_2(a). \tag{16}$$

If (16) is satisfied, for every $t \in (\tilde{\alpha}, \tilde{\beta})$ *the matrix* $D = \operatorname{diag}(t, 1)$ *is a common solution.*

Proof. $\Rightarrow$: Assume that there exists a common diagonal solution and $\operatorname{diag} = (t_*, 1)$ is a such solution. Then for any $a \in Q$

$$Et_*^2 - Ft_* + Q < 0. \tag{17}$$

If $a \in Q_1$, then $\dfrac{F - \sqrt{\Delta}}{2E} = \dfrac{2G}{F + \sqrt{\Delta}} < t_* < \dfrac{F + \sqrt{\Delta}}{2E}$,

If $a \in Q_2$, then $E(a) = 0$ and $\dfrac{G}{F} = \dfrac{2G}{F + \sqrt{\Delta}} < t_* < \infty$. Recall that the sets Q_1 and Q_2 are defined by (11).

Consequently, for any $a \in Q$

$$\tilde{x}_1(a) < t_* < \tilde{x}_2(a),$$

where $\tilde{x}_1(a)$ and $\tilde{x}_2(a)$ are defined by (15).

The function $\tilde{x}_1(a)$ is continuous in $a \in Q$. Therefore

$$\max_{a \in Q} \tilde{x}_1(a) < t_* \leq \inf_{a \in Q} \tilde{x}_2(a),$$

and (16) is satisfied.

$\Leftarrow$: Conversely, assume that (16) is satisfied and $t_* \in (\tilde{\alpha}, \tilde{\beta})$. Then for all $a \in Q$ we have $\tilde{x}_1(a) < t_* < \tilde{x}_2(a)$ or

$$\frac{2G}{F + \sqrt{\Delta}} < t_* < \tilde{x}_2(a). \tag{18}$$

If $a \in Q_1$, then $E > 0$ and from (18) we obtain

$$\frac{F - \sqrt{\Delta}}{2E} < t_* < \frac{F + \sqrt{\Delta}}{2E} \quad \text{or} \quad Et_*^2 - Ft_* + G < 0.$$

If $a \in Q_2$, then $E = 0$ and (18) gives

$$\frac{2G}{F + \sqrt{\Delta}} = \frac{G}{F} < t_* < \infty,$$

or $Et_*^2 - Ft_* + G = -Ft_* + G < 0$. Consequently for all $a \in Q$

$$Et_*^2 - Ft_* + Q < 0$$

and $\mathrm{diag}(t_*, 1)$ is a common solution. □

Example 3. *Consider the following family*

$$\mathcal{A} = \begin{bmatrix} [-3,-2] + j\,[1,2] & [-1,2] + j\,[-3,1] \\ [-2,-1] + j\,[0.5,1] & [-5,-4] + j\,[1,2] \end{bmatrix}.$$

Here $0 \in [a_2^-, a_2^+] = [-1,2]$, $0 \in [\tilde{a}_2^-, \tilde{a}_2^+] = [-3,1]$ *and the family is robust diagonally stable, since* $a_1^+ = -2 < 0$, $a_4^+ = -4 < 0$ *and* $\min_a (F^2 - 4EG) = 284 > 0$, $\min_a F = 22 > 0$. *Define* $\tilde{x}_1(a)$ *and* $\tilde{x}_2(a)$ *as in Theorem 4. Then*

$$\tilde{\alpha} = \max_{a \in Q} \tilde{x}_1(a) = 0.257385,$$
$$\tilde{\beta} = \inf_{a \in Q} \tilde{x}_2(a) = 2.238979$$

and $\tilde{\alpha} < \tilde{\beta}$ *and by Theorem 4 for any* $t \in (\tilde{\alpha}, \tilde{\beta})$ *the matrix* $D = \mathrm{diag}(t,1)$ *is common solution.*

In this example, both sufficient conditions from [15] are not satisfied. The matrix U *from Theorem 1 is* $U = \begin{bmatrix} -2 & \sqrt{13} \\ \sqrt{5} & -4 \end{bmatrix}$ *with* $\rho^H(U) = \max\{\mathrm{Re}(\lambda_i(U)) :\ i = 1,2\} = 0.010358 > 0$. *The inequality (5) is satisfied as well, indeed*

$$A_0 = \begin{bmatrix} -\frac{5}{2} & \frac{1}{2} \\ -\frac{3}{2} & -\frac{9}{2} \end{bmatrix}, \qquad \tilde{A}_0 = \begin{bmatrix} \frac{3}{2} & -1 \\ \frac{3}{4} & \frac{3}{2} \end{bmatrix},$$

$$R_A = \begin{bmatrix} \frac{1}{2} & \frac{3}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}, \qquad R_{\tilde{A}} = \begin{bmatrix} \frac{1}{2} & 2 \\ \frac{1}{4} & \frac{1}{2} \end{bmatrix},$$

$$\rho^H\left(\begin{bmatrix} A_0 & \tilde{A}_0 \\ -\tilde{A}_0 & A_0 \end{bmatrix}\right) + \rho^H\left(\begin{bmatrix} R_A & R_{\tilde{A}} \\ R_{\tilde{A}} & R_A \end{bmatrix}\right) = -2.25 + 2.620185 = 0.370185 > 0.$$

4. Conclusion

In this paper for a second order complex interval family we consider the problem of existence and evaluation of common diagonal solutions to the Lyapunov matrix inequalities. This problem is very important in the stability theory, since it gives more simple Lyapunov functions of the diagonal types. Firstly, a criterion for the robust diagonal stability is given. The existence problems of a diagonal solutions are reduced to the simple smooth optimization problems.

Obtaining smiliar conditions for a third order complex interval family and for a general complex interval familiy may serve the topics of the future investigations.

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